Problem 1. (10 points) For each of the following languages, determine whether it is:
R = recursive,
RE - R = recursively enumerable but not recursive,
coRE - R = the complement of a recursively enumerable language but not recursive, or
N = neither recursively enumerable nor the complement of a recursively enumerable language.

\{M; x \mid M(x) \neq \not\}\}
\{M; x \mid M(x) = \not\}\}
\{M; x \mid M(x) \text{ will not reach cell 100 of the tape}\}\}
\{M; x \mid M(x) \text{ halts after } \leq 100 \text{ steps}\}\}
\{M \mid M(\lambda) = \not\}\}
\{M_1; M_2 \mid L(M_1) = L(M_2)\}\}
\{M_1; M_2 \mid L(M_1) \neq L(M_2)\}\}
\{M; x \mid M(x) \text{ will reach cell 100 of the tape}\}\}
\{M; x \mid M(x) \neq \not, \text{ but takes more than } 100 \text{ steps}\}\}
\{M_1; M_2; M_3 \mid L(M_1) = L(M_2) \neq L(M_3)\}\}
\{M_1; M_2 \mid \exists x \text{ such that } M_1(x) = \not \text{ and } M_2(x) \neq \not\}\}
\{M \mid \forall x \text{ M(x) } \neq \not\}\}
\{M \mid \exists x \text{ M(x) } \neq \not\}\}
\{M; x \mid M(x) \text{ takes more than } 100 \text{ steps}\}\}
\{M \mid \exists x \text{ such that the 3rd transition of } M(x) \text{ is to the right}\}\}
\{M \mid M(M) = \not\}\}
\{M \mid M(M) \neq \not\}\}

Problem 2. (10 points) Assume the graph on the left shows a network (with capacities) and the
graph on the right shows the current flow in the network, construct the derived network and find an
augmenting path.

Problem 3. (10 points) Prove that
L = \{M \mid \exists x \text{ such that } M(x) \neq \not\}\} is not recursive.

Problem 4. (10 points) Prove that
L = \{M \mid \forall x \text{ M(x) } \neq \not\}\} is not recursive.
Problem 5. (10 points) Prove that $L = \{ M : \exists x, y \text{ such that } M(x) \neq \bot \text{ and } M(y) \neq \bot \}$ is not recursive.

Problem 6. (10 points) Prove that the language below is NP-complete by reducing CLIQUE to $L$.

$L = \{ G; k; l : \exists a set S of k vertices each of which is connected to at least \ell vertices in S \}$

Problem 7. (10 points) Prove that the language below is NP-complete by reducing HAMILTONIAN CYCLE to $L$.

$L = \{ G; k : G \text{ contains a cycle with } k \text{ vertices} \}$

Problem 8. (10 points) Prove that the language below is NP-complete by reducing INDEPENDENT SET to $L$.

$L = \{ G; k; l : \exists a set S of k vertices each of which is connected to at most \ell vertices in S \}$

Problem 9. (10 points) Prove that the language below is NP-complete by reducing CLIQUE to $L$.

$L = \{ G_1; G_2; k : \text{both } G_1 \text{ and } G_2 \text{ have a clique of size } k \}$

Problem 10. (10 points) Prove that the language below is NP-complete by reducing HAMILTONIAN CYCLE to $L$.

$L = \{ G_1; G_2 : \text{both } G_1 \text{ and } G_2 \text{ have a hamiltonian cycle} \}$

Problem 11. (10 points) Prove that the language below is NP-complete by reducing INDEPENDENT SET to $L$.

$L = \{ G_1; G_2; k : \text{both } G_1 \text{ and } G_2 \text{ have independent sets of size } k \}$