COMP610 Midterm Review Questions

Problem 1. (10 points) For each of the following languages, determine whether it is:
R = recursive,
RE - R = recursively enumerable but not recursive,
coRE - R = the complement of a recursively enumerable language but not recursive, or
N = neither recursively enumerable nor the complement of a recursively enumerable language.

$\{M; x \ : \ M(x) \neq \nearrow\}$ __________

$\{M; x \ : \ M(x) = \nearrow\}$ __________

$\{M; x \ : \ M(x) \ \textit{will not reach cell } 100 \textit{ of the tape}\}$ __________

$\{M; x \ : \ M(x) \ \textit{halts after } \leq 100 \textit{ steps}\}$ __________

$\{M \ : \ M(\lambda) = \nearrow\}$ __________

$\{M_1; M_2 \ : \ L(M_1) = L(M_2)\}$ __________

$\{M_1; M_2 \ : \ L(M_1) \neq L(M_2)\}$ __________

$\{M; x \ : \ M(x) \ \textit{will reach cell } 100 \textit{ of the tape}\}$ __________

$\{M; x \ : \ M(x) \neq \nearrow, \textit{ but takes more than } 100 \textit{ steps}\}$ __________

$\{M_1; M_2; M_3 \ : \ L(M_1) = L(M_2) \neq L(M_3)\}$ __________

$\{M_1; M_2 \ : \ \exists \ x \textit{ such that } M_1(x) = \nearrow \textit{ and } M_2(x) \neq \nearrow\}$ __________

$\{M_1; M_2 : M_1(\lambda) = \nearrow \textit{ and } M_2(\lambda) \neq \nearrow\}$ __________

$\{M \ : \ \forall \ x \ M(x) \neq \nearrow\}$ __________

$\{M \ : \ \exists \ x \ M(x) \neq \nearrow\}$ __________

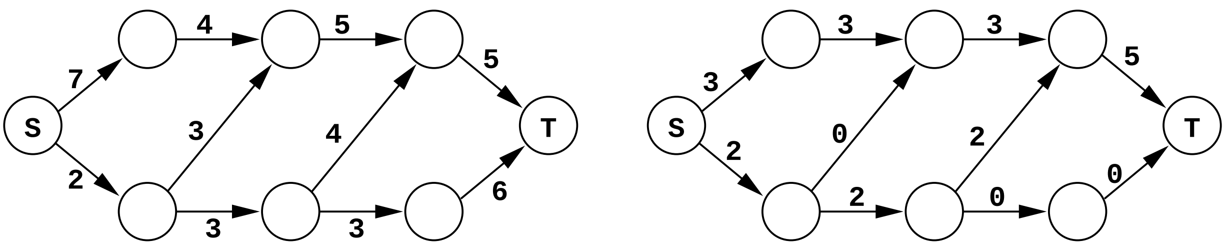
$\{M; x \ : \ M(x) \textit{ takes more than } 100 \textit{ steps}\}$ __________

$\{M \ : \ \exists \ x \textit{ such that the 3rd transition of } M(x) \textit{ is to the right}\}$ __________

$\{M \ : \ M(M) = \nearrow\}$ __________

$\{M \ : \ M(M) \neq \nearrow\}$ __________

Problem 2. (10 points) Assume the graph on the left shows a network (with capacities) and the graph on the right shows the current flow in the network, construct the derived network and find an augmenting path.



Problem 3. (10 points) Prove that $L = \{M \ : \ \exists \ x \textit{ such that } M(x) \neq \nearrow\}$ is not recursive.

Problem 4. (10 points) Prove that $L = \{M \ : \ \forall \ x \ M(x) \neq \nearrow\}$ is not recursive.

Problem 5. (10 points) Prove that $L = \{M \ : \ \exists\, x, y \textit{ such that } M(x) \neq \nearrow \ \&\ M(y) \neq \nearrow\}$ is not recursive.

Problem 6. (10 points) Prove that the language below is NP-complete by reducing **CLIQUE** to L.

$$L = \{G; k; l \ : \ \exists \textit{ a set } S \textit{ of } k \textit{ vertices each of which is connected to at least } \ell \textit{ vertices in } S\}$$

Problem 7. (10 points) Prove that the language below is NP-complete by reducing **HAMILTONIAN CYCLE** to L.

$$L = \{G; k \ : \ G \textit{ contains a cycle with } k \textit{ vertices}\}$$

Problem 8. (10 points) Prove that the language below is NP-complete by reducing **INDEPENDENT SET** to L.

$$L = \{G; k; l \ : \ \exists \textit{ a set } S \textit{ of } k \textit{ vertices each of which is connected to at most } \ell \textit{ vertices in } S\}$$

Problem 9. (10 points) Prove that the language below is NP-complete by reducing **CLIQUE** to L.

$$L = \{G_1; G_2; k \ : \ \textit{both } G_1 \textit{ and } G_2 \textit{ have a clique of size } k\}$$

Problem 10. (10 points) Prove that the language below is NP-complete by reducing **HAMILTONIAN CYCLE** to L.

$$L = \{G_1; G_2 \ : \ \textit{both } G_1 \textit{ and } G_2 \textit{ have a hamiltonian cycle}\}$$

Problem 11. (10 points) Prove that the language below is NP-complete by reducing **INDEPENDENT SET** to L.

$$L = \{G_1; G_2; k \ : \ \textit{both } G_1 \textit{ and } G_2 \textit{ have independent sets of size } k\}$$