

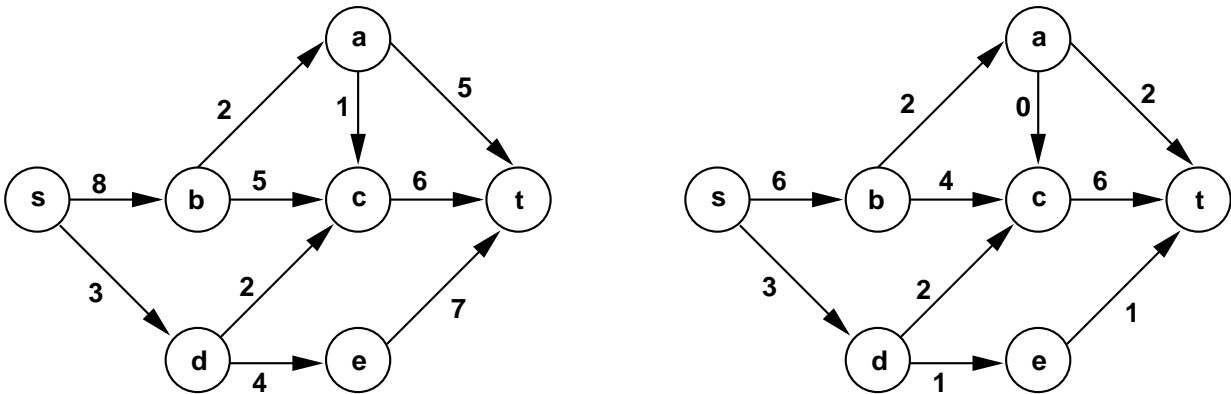
COMP610 Midterm - Solution Sketches

Problem 1. (10 points) Label each of the following statements TRUE or FALSE.

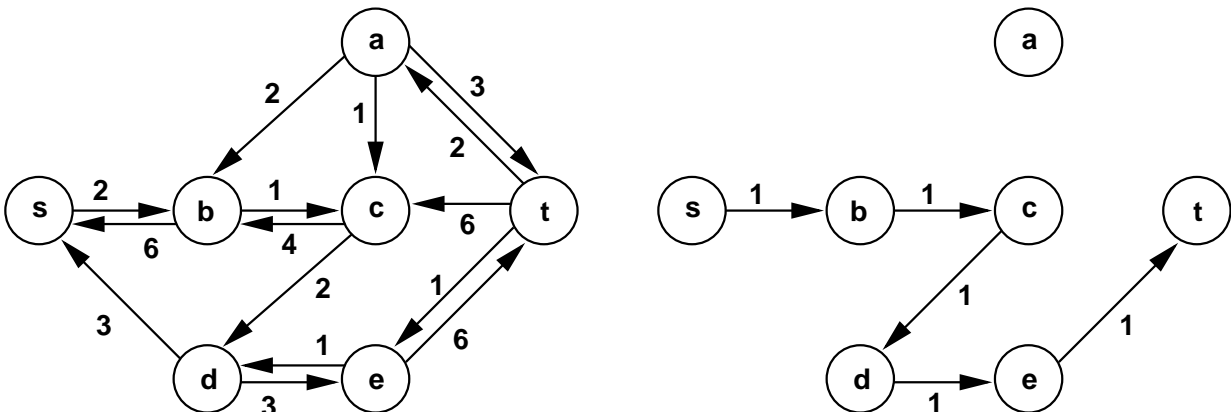
Every directed graph with n vertices and $n(n - 1)/2$ edges has a directed cycle.	<u>FALSE</u>
For any network the maximum cut is equal to the maximum flow.	<u>FALSE</u>
The set of languages accepted by 2-tape Turing Machines is the same as the set of languages accepted by 1-tape Turing Machines.	<u>TRUE</u>
Every Turing Machine accepts a recursively enumerable language.	<u>TRUE</u>
HALTING is a recursive language.	<u>FALSE</u>
Every recursive language is recursively enumerable.	<u>TRUE</u>
\exists a recursively enumerable language whose complement is also recursively enumerable.	<u>TRUE</u>
If there is a reduction from PROB to HALTING then PROB must be undecidable.	<u>FALSE</u>
BIPARTITEMATCHING is NP-complete.	<u>FALSE</u>
There is a polynomial time reduction from 3SAT to VERTEXCOVER.	<u>TRUE</u>

Answer 1. F, F, T, T, F, T, T, F, F, T

Problem 2. (10 points) Assume the graph on the left shows a network (with capacities) and the graph on the right shows the current flow in the network, construct the derived network and find an augmenting path.



Answer 2. The graph on the left is the derived network. The path on the right is the augmenting path.



Problem 3. (10 points) Show that the problem below is not decidable.

$$L = \{(M;w): M(w) \text{ halts after an even number of steps}\}$$

Answer 3. Given an input to HALTING $(M;w)$ create an input to L $(M';w')$ where $w' = w$ and M' (on any input) simulates $(M;w)$ and keeps track of the parity of the number of steps and (if it terminates) then either immediately halts or performs one final step and halts. M' chooses whichever of these causes the number of steps to be even.

Problem 4. (10 points) For each boolean expression either give a satisfying truth assignment or state that it is unsatisfiable.

a) $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_4}) \wedge (\overline{x_2} \vee \overline{x_3} \vee x_4)$

b) $((\overline{x_1} \wedge x_2) \vee x_3) \wedge ((\overline{x_1} \vee \overline{x_2}) \wedge x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee \overline{x_3}) \wedge ((\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_4}) \vee (\overline{x_2} \wedge \overline{x_3} \wedge x_4))$

c) $(x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_4}) \wedge (\overline{x_2} \vee \overline{x_3} \vee x_4)$

Answer 4.

a) $x_1 = T, x_2 = F, x_3 = F, x_4 = T$ is a satisfying truth assignment.

b) No satisfying truth assignment.

x_1	x_2	x_3	x_4	expression
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

c) $x_1 = T, x_2 = F, x_3 = F, x_4 = T$ is a satisfying truth assignment.

Problem 5. (10 points) Recall that SUBSETSUM is NP-complete. Show that the language EQUAL is also NP-complete.

$$\text{EQUAL} = \{ S : S \text{ is a set of integers which can be split into two sets which sum to the same value} \}$$

Answer 5. Given an instance (S, t) of SUBSETSUM where S is a set of numbers and t is the target create an instance of EQUAL by setting $S' = S + \{s\}$ where $s = |2t - \sum_S s_i|$.

If S has a subset that sums to exactly t then S can be split up into a set that adds up to t and a set that adds up to $\sum_S s_i - t$ by adding s to the smaller of these both sets have the same sum (this sum is either t or $\sum_S s_i - t$ depending on whether t is more or less than half the total).

If the S' created has two subsets that sum to exactly the same value then they either sum to t or $\sum_S s_i - t$. In the first case, the set that does not include s is a subset of S that sums to t . In the second case, removing s from the set that contains s results in a subset of S that sums to t .

Problem 6. (10 points) The BINPACKING problem asks whether there is a way to split a set S of integers into b sets each of which sums to at most t . Show that BINPACKING is NP-complete.

BINPACKING = $\{(S; b, t): S \text{ can be split into } b \text{ sets each of which sums to at most } t\}$

Answer 6. The previous problem showed that EQUAL was NP-complete. Given an S an instance of EQUAL create an instance of BINPACKING $(S; 2; t)$ where $t = \frac{\sum_S s_i}{2}$.

If S has two equal sized subsets then they can be put into two bins with capacity half the sum. If S doesn't have two equal sized subsets then they cannot be put into two bins with capacity half the sum.