Problem 1. (10 points) Label each of the following statements True or False.

Every directed graph with \( n \) vertices and \( n(n - 1)/2 \) edges has a directed cycle.  

For any network the maximum cut is equal to the maximum flow.  

The set of languages accepted by 2-tape Turing Machines is the same as the set of languages accepted by 1-tape Turing Machines.  

Every Turing Machine accepts a recursively enumerable language.  

HALTING is a recursive language.  

Every recursive language is recursively enumerable.  

\( \exists \) a recursively enumerable language whose complement is also recursively enumerable.  

If there is a reduction from PROB to HALTING then PROB must be undecidable.  

BIPARTITEMATCHING is NP-complete.  

There is a polynomial time reduction from 3SAT to VERTEXCOVER.
Problem 2. (10 points) Assume the graph on the left shows a network (with capacities) and the graph on the right shows the current flow in the network, construct the derived network and find an augmenting path.
Problem 3. (10 points) Show that the problem below is not decidable.

\[ L = \{ (M;w) : M(w) \text{ halts after an even number of steps} \} \]
Problem 4. (10 points) For each boolean expression either give a satisfying truth assignment or state that it is unsatisfiable.

a) \((x_1 \lor x_2 \lor x_3) \land (\overline{x_1} \lor x_2 \lor x_3) \land (\overline{x_1} \lor x_2 \lor \overline{x_3}) \land (x_1 \lor \overline{x_2} \lor x_3) \land (\overline{x_1} \lor x_2 \lor \overline{x_4}) \land (x_2 \lor \overline{x_3} \lor x_4)\)

b) \(((x_1 \land x_2) \lor x_3) \land ((\overline{x_1} \lor \overline{x_2}) \land x_3) \land (x_1 \lor x_2 \lor \overline{x_3}) \land (\overline{x_1} \lor x_2 \lor \overline{x_4}) \land ((x_1 \land \overline{x_2} \land \overline{x_4}) \lor (x_2 \land \overline{x_3} \land x_4))\)

c) \((x_1 \lor x_2) \land (x_1 \lor x_2 \lor x_3) \land (\overline{x_1} \lor x_2 \lor x_3) \land (x_1 \lor x_2 \lor \overline{x_3}) \land (\overline{x_1} \lor x_2 \lor \overline{x_4}) \land (x_2 \lor \overline{x_3} \lor x_4)\)
Problem 5. (10 points) Recall that \textsc{SubsetSum} is NP-complete. Show that the language \textsc{Equal} is also NP-complete.

\textsc{Equal} = \{ \text{S : S is a set of integers which can be split into two sets which sum to the same value} \}
Problem 6. (10 points) The BinPacking problem asks whether there is a way to split a set $S$ of integers into $b$ sets each of which sums to at most $t$. Show that BinPacking is NP-complete.

$$\text{BinPacking} = \{(S; b, t): S \text{ can be split into } b \text{ sets each of which sums to at most } t\}$$