

COMP 610 Homework 3 - Solution Sketches

1. Solve the following instances of the Post Correspondence Problem.

- $\{(001, 10), (110, 01), (01, 110), (10, 100)\}$
- $\{(11, 0), (01, 1), (110, 1), (0, 00), (11, 010)\}$
- $\{(0, 1), (0, 001), (01, 1), (10, 0)\}$
- $\{(100, 1), (1, 111), (110, 0), (1, 110)\}$
- $\{(110, 1), (1, 01), (0, 110)\}$

Answers:

- No solution. None of the 4 items can be the last in the sequence (don't match).
- 4, 2. Both yield 001.
- 2, 3, 4. Both yield 00110.
- 1, 3, 3, 4, 4, 2. Both yield 100110110111.
- No idea (I mentioned that there was a hard one).

2. Which of the following boolean expressions are satisfiable / unsatisfiable.

- $(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee \overline{x_4}) \wedge (\overline{x_1} \vee \overline{x_4} \vee x_5) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_5}) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_5}) \wedge (\overline{x_1} \vee \overline{x_4} \vee x_5) \wedge (\overline{x_1} \vee x_5 \vee \overline{x_6}) \wedge (\overline{x_4} \vee x_5 \vee x_6) \wedge (\overline{x_3} \vee x_4 \vee \overline{x_5}) \wedge (\overline{x_1} \vee x_3 \vee x_6) \wedge (\overline{x_3} \vee x_5 \vee \overline{x_6}) \wedge (\overline{x_1} \vee x_2 \vee x_6)$
- $(x_1 \vee \overline{x_4} \vee x_2) \wedge (x_2 \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_1} \vee \overline{x_3} \vee x_2) \wedge (x_1 \vee \overline{x_2} \vee x_4) \wedge (x_3 \vee \overline{x_1} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_4) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_4} \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_4} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_2} \vee \overline{x_4} \vee \overline{x_3}) \wedge (\overline{x_2} \vee x_3 \vee x_4) \wedge (\overline{x_1} \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_4} \vee \overline{x_2} \vee x_1) \wedge (x_1 \vee x_2 \vee x_4) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_4})$
- $(\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_1} \vee \overline{x_4} \vee x_2) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge (\overline{x_3} \vee x_4 \vee x_1) \wedge (\overline{x_1} \vee x_3 \vee x_2) \wedge (\overline{x_2} \vee x_3 \vee x_4) \wedge (\overline{x_4} \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2 \vee x_4) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_3})$

Answers:

- Satisfiable by making all variables false.
- Unsatisfiable.
- Satisfiable by making all variables false.

3. Solve the following 3 instances of SubsetSum. $S = \{23, 29, 39, 44, 50, 57, 63, 71, 73, 81, 99, 104\}$, $T_1 = 361$, $T_2 = 364$, $T_3 = 390$.

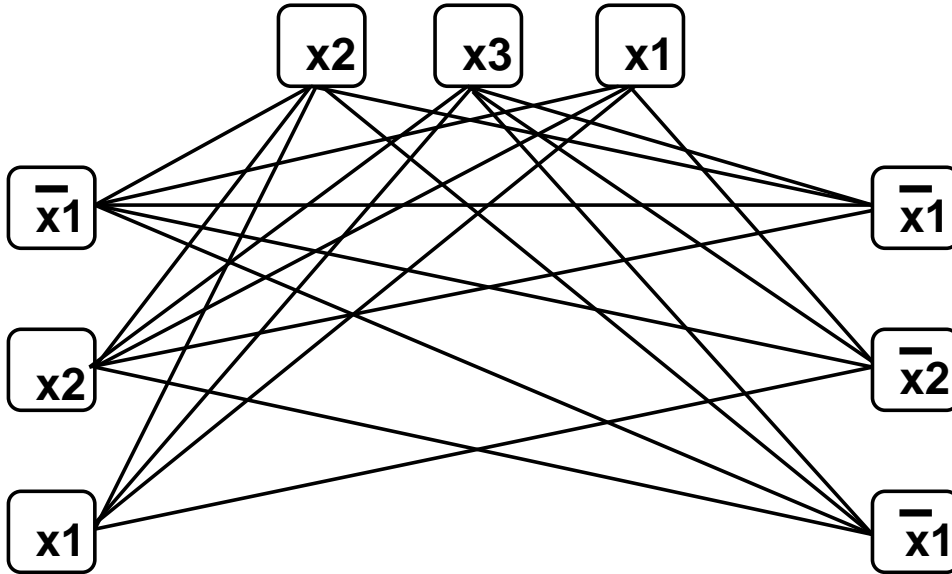
Answers:

- $23+29+44+50+63+71+81 = 361$.
- $23+39+44+73+81+104 = 364$
- $29+39+50+57+63+71+81 = 390$

4. Perform the reduction from 3SAT to CLIQUE for the following boolean expression:

$$(x_1 \vee x_2 \vee \overline{x_1}) \wedge (x_2 \vee x_3 \vee x_1) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_1}).$$

For every literal in every clause create a vertex. Connect any two vertices if their literals do not contradict.



5. Perform the reduction from 3SAT to SUBSETSUM for the following boolean expression:

$$(x_1 \vee x_2 \vee \bar{x}_1) \wedge (x_2 \vee x_3 \vee x_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_1) \wedge (x_1 \vee x_2 \vee \bar{x}_3).$$

An instance of SUBSETSUM is a set of integers and a target. Following the steps of the reduction yields:
 $S = \{100110, 100101, 010110, 010001, 001010, 001000, 000100, 000100, 000010, 000010, 000001, 000001\}$,
 $T = 111333$.

6. Show that *Dom2* is NP-complete.

$$\text{Dom2} = \{G; k : G \text{ has a set of vertices } S \text{ with size } k \text{ such that every vertex in } G \text{ is at most distance } 2 \text{ from some vertex in } S\}$$

We will reduce Dominating Set to Dom2. Given an instance of Dominating Set $(G; k)$ we create an instance of Dom2 $(G'; k')$. To create G' begin with G and for every vertex v add an additional vertex v' and connect v' to v . Let $k'=k$.

If G has a dominating set of size k then this same set will be a set of k vertices which is at most one away from any of the original vertices and therefore at most 2 away from each new vertex.

If G' has a set S consisting of k vertices for which every vertex is within distance 2 then we claim that G has a set of k vertices for which every vertex is within distance 1. If S contains any of the newly created vertices replace them with the associated original vertices. This does not cause S to become larger and the only vertices for which the distance might increase are the newly created vertices (which may have had distances increase from 0 to 1). So S still fulfills the conditions of Dom2. Since both original and new vertices are within distance 2, every original vertex must be within distance 1 (otherwise its newly created vertex would be more than 2 away). So S is also a vertex cover of G .

7. Errol Iner is a traveling salesman. However, Errol is unusual for a traveling salesman because he likes to fly on airplanes. So when given a choice he selects to maximize his flying time. From time to time he is assigned a set of cities where he needs to travel to sell his product. Show that Errol's problem is difficult by providing a reduction from HAMILTONIANCYCLE to

$$L = \{G; k : G \text{ has a Hamiltonian Cycle with length at least } k\}.$$

Given an instance of Hamiltonian Cycle, G , create an instance of L , $(G'; k)$, by labeling every existing edge with 2, adding all missing edges and labeling them with 1, and setting $k=2n$ (where n is the number of vertices).

If G has a Hamiltonian Cycle then this same cycle will have length k in G' .

If G does not have a Hamiltonian cycle then any Hamiltonian cycle in G' must contain at least one edge labeled 1. So the length is less than k .