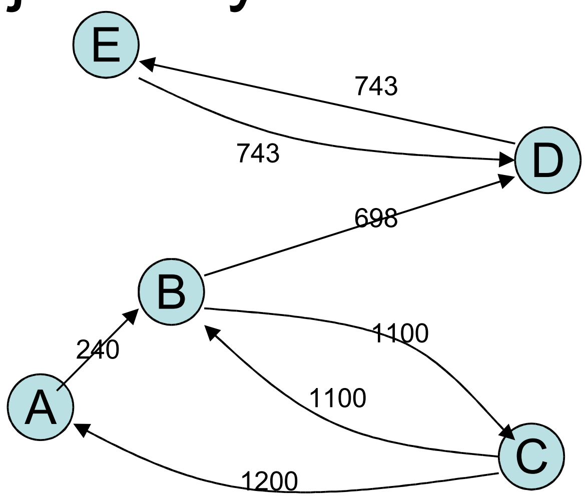
COMP282
Advanced Data Structures

Lecture 04
Graphs
Implementation and Traversal

Implementation

- How do we represent graphs with a computer?
- Adjacency Matrix:



	A	B	C	D	E
A	0	240	0	0	0
B	0	0	1100	698	0
C	1200	1100	0	0	0
D	0	0	0	0	743
E	0	0	0	743	0

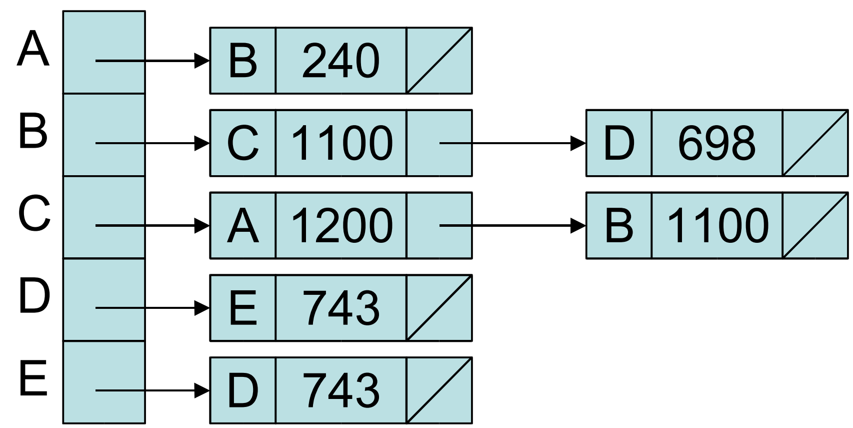
	A	B	C	D	E
A	∞	240	∞	∞	∞
B	∞	∞	1100	698	∞
C	1200	1100	∞	∞	∞
D	∞	∞	∞	∞	743
E	∞	∞	∞	743	∞

Multidimensional Array based

- Adjacency matrix:
  - Advantages:
    - Easy to maintain.
    - Determination if an edge exists between two vertices is simple and efficient.
  - Disadvantages:
    - Might be difficult to avoid representing absent nodes
      - 0 might be a valid edge and infinity is not valid
    - Consume a lot of space. Especially for disconnected/non-complete or sparse graphs.
    - Might not be possible due to number of Nodes. Memory consumed is proportional to the square of the number of nodes. 100,000 nodes would require more memory than a 32-bit addressable machine could provide.

Reference based

- Adjacency List:



- Disadvantages
  - Complicated to administer and maintain
  - Less efficient at determining if edges exist.
- Advantages
  - Great for sparsely connected graphs.
  - Efficient at determining which vertices are connected to a particular vertex.

Rules based

- Some problems present graphs that are too large to store in their entirety.
  – Chess and Go are two such examples
- Instead of storing actually edges we can store, code or otherwise implement rules that define what is connected to what.
  – In chess we know that one board state is adjacent to another if we can move from one to other with a single, well defined, valid move.
- Each graph node then is an object that contains:
    - A reference to a object that contains the label, state or information for the node.
    - A linked list of references to other nodes (these are the edges.
- In any case it is important to choose your implementation wisely on the basis of the project and goal's requirements and your available resources.

Traversing Graphs

- It is often necessary to visit all the nodes in a graph. We therefore desire algorithms capable of traversing all the nodes of a graph.
- Similar to pre-order, in-order traversals of BSTs
- There are two distinct ways of thinking about graph traversal:
  - Depth first
  - Breadth first

Connected Components

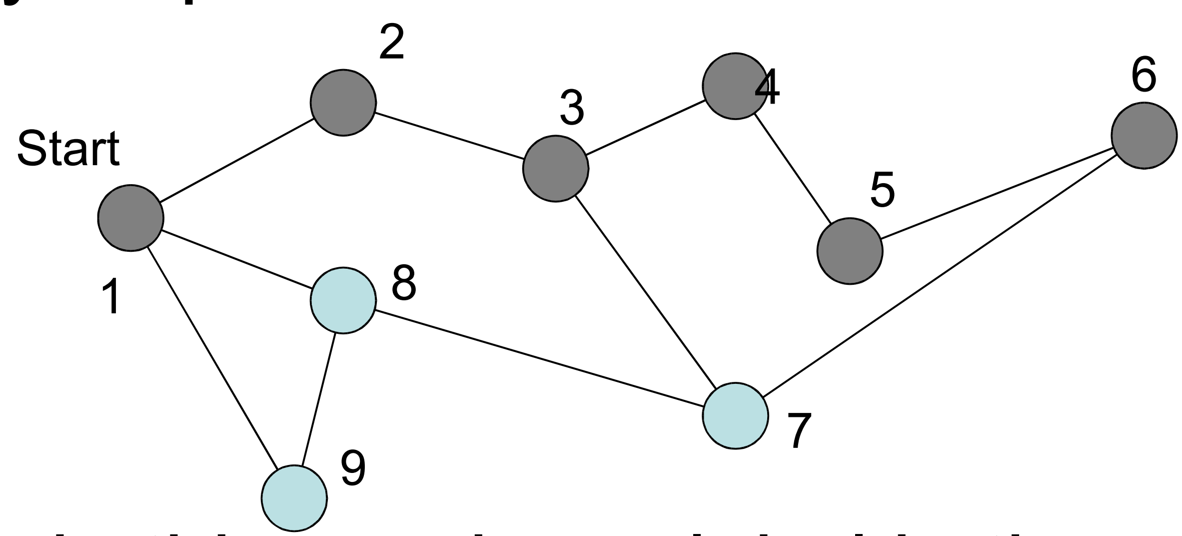
- Only guaranteed to visit all the nodes if the graph is connected. (there exists a path from any vertex to any other vertex.)
- If a graph isn't connected these traversals will only traverse those nodes that are connected to the starting node *v*.
- More terminology: The subset visited is termed the "connected component" containing *v.*

Cycles

- If a graph contains cycles then it is possible for simple traversal algorithms to loop indefinitely; visiting the same node(s) repeatedly.
- To prevent this nodes are “marked” when they are first visited and the traversal never visits a marked node.
  - Nodes can be marked by setting true in an array or by the presence of the Node as a key in some Hash or Tree map.

Depth First

- The concept behind depth first search is to visit nodes as “deeply” into the graph as quickly as possible:



- Nodes in this graph are label in the order they are visited by a Depth First Traversal

Recursive DFS

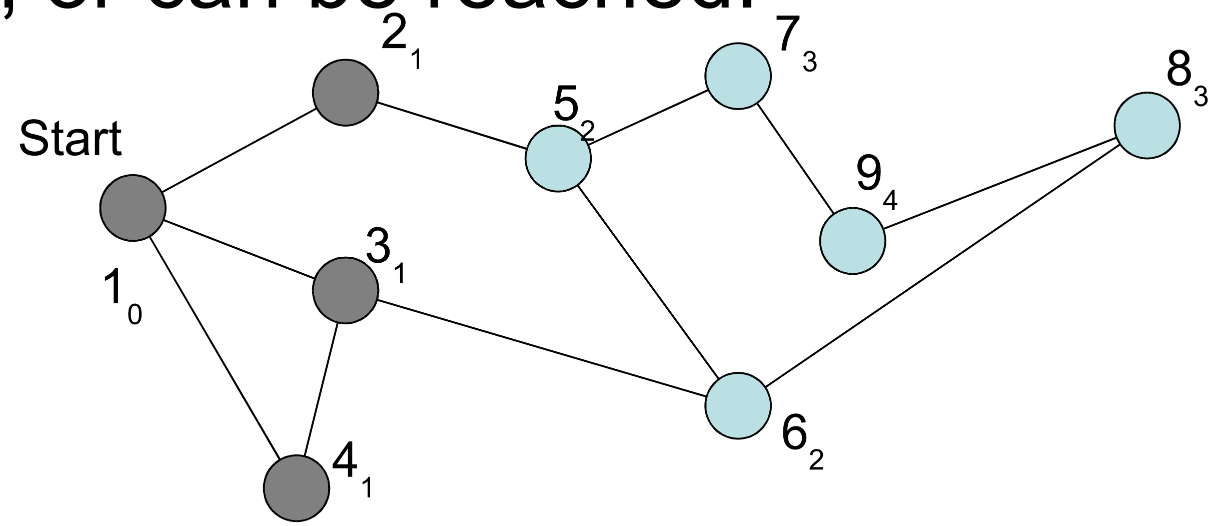
- Algorithm:
- ```
  Dfs(v)
  {
     mark v as visited;
     for (each unvisited vertex u adjecent to v)
         dfs(u)
  }
  ```
  ```

Iterative DFS

- ```
  Dfs(v)
  {
     Stack s = new Stack();
     s.push(v);
     mark v as visited;
     while (!s.isEmpty())
        if (s.peek() has no unvisited, adjacent nodes)
           s.pop(); // done with node on top
        else {
           u = select unvisited, adjacent node to s.peek();
           s.push(u);
           mark u as visited;
        }
  }
  ```
  ```

Breadth First Search

- The concept behind breadth first search is that nodes are visited as soon as they are found, or can be reached:



- Nodes are now labeled with the order visited and the minimum distance from the start.

Iterative BFS

- ```
  Bfs(v)
  {
     Queue q = new Queue();
     s.enqueue(v);
     mark v as visited;
     while (!s.isEmpty()) {
        w = q.dequeue(); // working on nodes reachable
                         // from node w
        // do any processing of w needed.
        for (each unvisited node u adjacent to w) {
           mark u as visited
           q.enqueue(u);
        }
     }
  }
  ```
  ```