BIBLIOGRAPHY

JORGE BALBÁS

1. Balbás J., Tadmor E., and Wu C.C.: (2004): *Central non-oscillatory schemes for one- and two-dimensional MHD equations. I.* J. Comput. Physics., 201(1):261-285.
2. Balbás J., Tadmor E.: (2005): *A central differencing simulation of the Orszag-Tang vortex system.* IEEE Transactions on Plasma Science, The 4th Triennial Special Issue on Images in Plasma Science 33(2) 470-471.
3. Balbás J., Tadmor E.: (2006): *Central non-oscillatory schemes for one- and two-dimensional MHD equations. II: high-order semi-discrete schemes.* SIAM J. Sci. Comput., *to appear*.
4. Balbás J., Karni S.: (2006): *A non-oscillatory central scheme for one-dimensional shallow water flows along channels with irregular geometry. under review*.

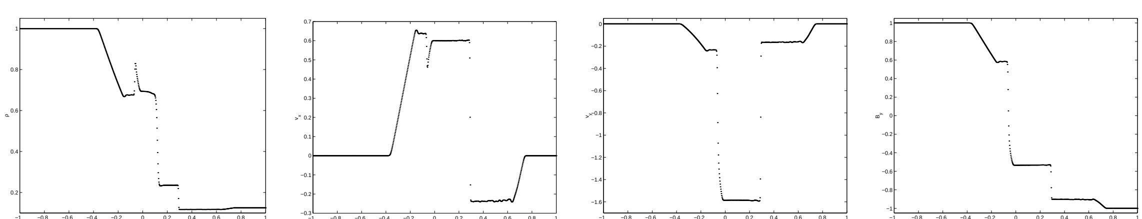
Thesis

1. Balbás, J.: (2004): *Non-oscillatory central schemes for the equations of magnetohydrodynamics in one- and two-space dimensions*, Ph.D. Thesis, University of California, Los Angeles.

Abstracts

1. **Central Non-oscillatory Schemes for One- and Two-dimensional MHD equations. I.**: (2004): (with E. Tadmor and C.C. Wu) J. Comput. Physics., 201(1):261-285, 2004.

   In this paper we utilize a family of high-resolution, non-oscillatory central schemes for the approximate solution of the equations of ideal magnetohydrodynamics (MHD) in one- and two-space dimensions. We present several prototype problems. Solutions of one-dimensional shock-tube problems is carried out using second- and third-order central schemes of [Numer. Math. 79 (1998) 397; J. Comput. Phys. 87 (2) (1990) 408] , and we use the second-order central scheme of [SIAM J. Sci Comput. 19 (6) (1998) 1892] which is adapted for the solution of the two-dimensional Kelvin-Helmholtz and Orszag-Tang problems. A qualitative comparison reveals an excellent agreement with previous results based on upwind schemes. Central schemes, however, require little knowledge about the eigen-structure of the problem — in fact, we even avoid an explicit evaluation of the corresponding Jacobians, while at the same time they eliminate the need for dimensional splitting. The one- and two-dimensional computations reported in this paper demonstrate the remarkable versatility of central schemes as black-box, Jacobian-free MHD solvers.



(a) (b) (c) (d)

FIGURE 1. Results of Brio-Wu one-dimensional MHD shock tub computed with Jacobian free form of 3rd order Liu-Tadmor fully-discrete scheme. (a) density, (b) x-velocity, (c) y-velocity, (d) y-magnetic field

2. **A Central Differencing Simulation of the Orszag-Tang Vortex System.** (with E. Tadmor): IEEE Transactions on Plasma Science, The 4th Triennial Special Issue on Images in Plasma Science 33(2) 470-471, 2005.

   The Orszag-Tang vortex system describes the transition to supersonic turbulence for the equations of Magnetohydrodynamics (MHD) in two space dimensions. The complex interaction between various shock waves traveling at different speed regimes that characterizes the solution of this test problem requires the use of numerical schemes capable of detecting and resolving accurately steep gradients while avoiding the onset of spurious oscillations. A simulation of the Orszag-Tang MHD vortex system computed with a third-order semi-discrete central scheme is presented below. The central differencing approach avoids any detailed knowledge of the characteristic structure of the hyperbolic model, resulting in simple to implement, yet robust, *black-box* numerical schemes

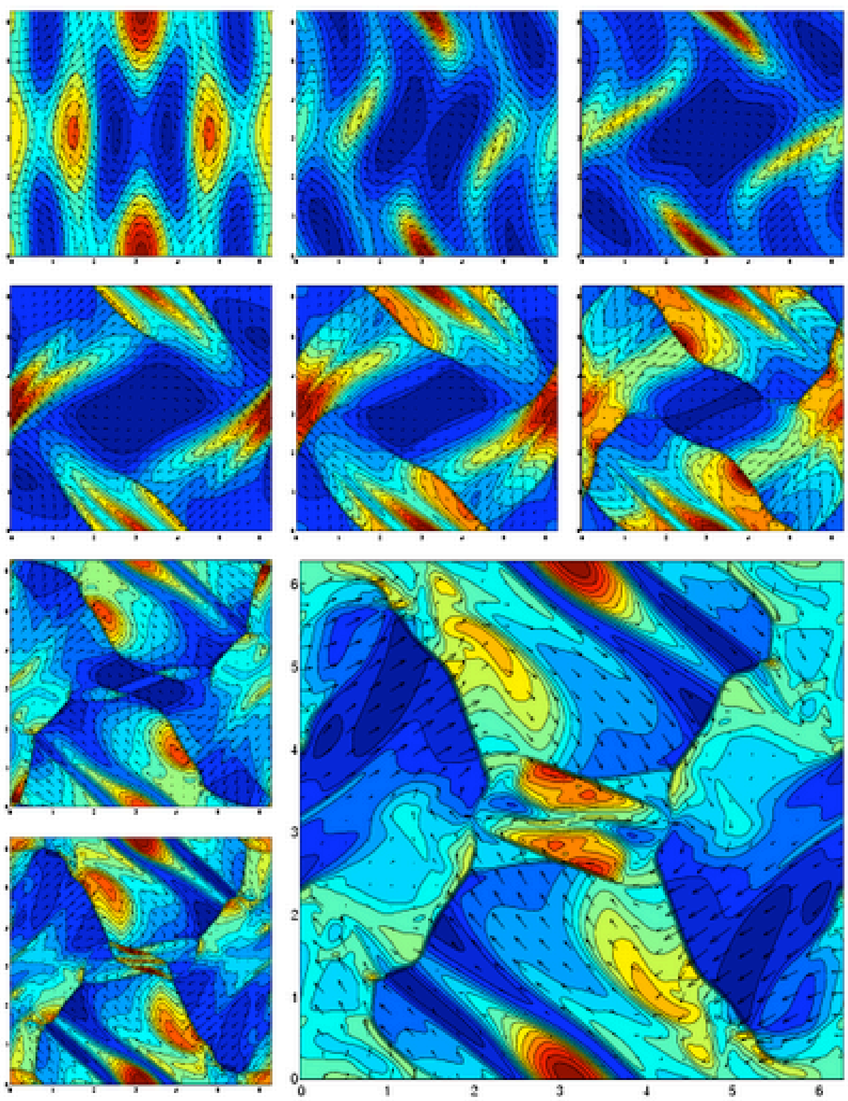


FIGURE 2. Evolution of density contours with superimposed xy-velocity field, from left to right and top to bottom: density contours from $t = 0.003$ –top-left corner– to $t = 2.7$ –lower-left corner– displayed at (approximately) 0.375 time intervals. The large image in the lower-right corner displays the solution at $t = 3$.

3. **Central Non-oscillatory Schemes for One- and Two-dimensional MHD Equations. II: High-order Semi-discrete Schemes.** (with E. Tadmor): SIAM J. Sci. Comput., *to appear*, 2006.

   We present a new family of high-resolution, non-oscillatory semi-discrete central schemes for the approximate solution of the ideal Magnetohydrodynamics (MHD) equations. This is the second part of our work, where we are passing from fully-discrete staggered schemes to the semi-discrete formulation advocated by Kurganov and Tadmor. This semi-discrete formulation retains the simplicity of fully-discrete central schemes while enhancing efficiency and adding versatility. The semi-discrete algorithm offers a wider range of options to implement its two key steps: non-oscillatory reconstruction of point values followed by the evolution of the corresponding point valued fluxes. We present the solution of several prototype MHD problems. Solutions of one-dimensional Brio-Wu shock-tube problems and the two-dimensional Kelvin-Helmholtz instability, Orszag-Tang vortex system, and the disruption of a high density cloud by a strong shock are carried out using third- and fourth-order central schemes based on the CWENO reconstructions. These results complement those presented in part I of our work and confirm the remarkable versatility and simplicity of central schemes as black-box, Jacobian-free MHD solvers. Furthermore, our numerical experiments demonstrate that this family of semi-discrete central schemes preserves the $\nabla \cdot \mathbf{B} = 0$-constraint within machine round-off error; happily, no constrained-transport enforcement is needed.

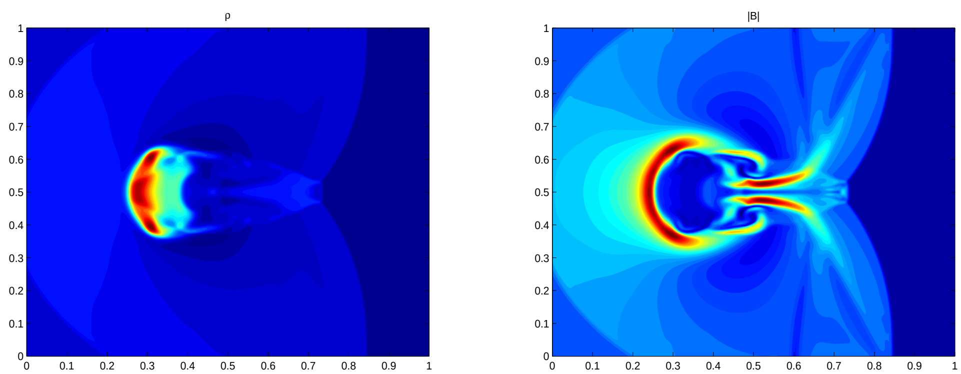


FIGURE 3. Third-order solution of the interaction between a strong shock and a high density cloud obtained with a 256×256 uniform grid at $t = 0.06$. Left: density, right: magnetic field, $|B|$.

4. **A Non-oscillatory Central Scheme for One-dimensional Shallow Water Flows along Channels with Irregular Geometry.** (with S. Karni): *under review*, 2006.

   We present a new high-resolution, non-oscillatory semi-discrete central scheme for one-dimensional shallow water flows along channels with non-uniform rectangular cross sections and bottom topography. The scheme extends existing central semi-discrete schemes for hyperbolic conservation laws and it enjoys two properties crucial for the accurate simulation of shallow water flows: it preserves the positivity of the water height, and it is well balanced, *i.e.*, the source terms arising from the geometry of the channel are discretized so as to balance the non-linear hyperbolic fluxes –a condition necessary for approximating correctly steady-state solutions. Along with a detailed description of the scheme and proofs of these two properties and an entropy inequality for the balance law, we present several numerical experiments –including the approximation of exact equilibrium solutions– that demonstrate the robustness –and simplicity– of the numerical algorithm.

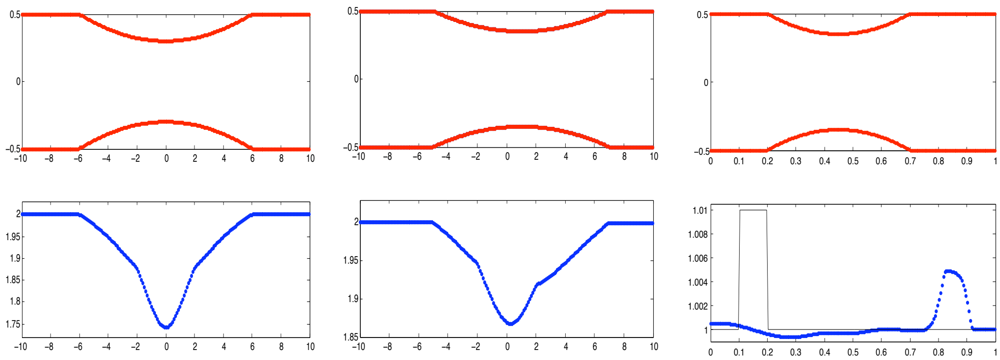


FIGURE 4. Steady-state solutions of shallow water flows. Top figures channel width, bottom figures total water height. Left: smooth flow through a channel with a centered contraction and bottom topography. Center: smooth flow through a channel with same contraction shifted to the right. Right: flow of a perturbation from steady-state of rest through a contraction shifted to the left – black line represents initial perturbation.

Thesis Abstract

Non-oscillatory Central Schemes for the Equations of Magnetohydrodynamics in One- and Two-space dimensions

Ph.D., Department of Mathematics, UCLA, June 2004
Thesis Advisor: Eitan Tadmor

Numerical schemes based on central differencing offer a universal approach for computing the solutions of nonlinear hyperbolic conservation laws in the sense that they are not tied to the characteristic structure of the underlying PDE, this allows for the straight forward implementation of these as *black-box* type numerical solvers. In this work we present a collection of high resolution, non-oscillatory central schemes for the equations of *Ideal Magnetohydrodynamics* (MHD) in one- and two-space dimensions.

We first introduce a family of fully-discrete central schemes based on the evolution of cell averages over a staggered grid. From this fully-discrete approximation of cell averages, making use of the information provided by the speed of propagation of the MHD waves, we pass (in the limit $\Delta t \downarrow 0$) to the more versatile non-staggered semi-discrete formulation of Kurganov and Tadmor. The numerical solution of several MHD prototype problems presented below demonstrate the ability of central schemes to detect and resolve accurately the steep gradients that characterize the solutions of these equations. A qualitative comparison reveals an excellent agreement with previous results computed with schemes based on upwind differencing. Central schemes, however, require little knowledge about the eigen structure of the problem — in fact, these schemes avoid the explicit evaluation of the corresponding Jacobians and, at the same time, eliminate the need for dimensional splitting in several space dimensions. The one- and two-dimensional computations reported in this work demonstrate the remarkable versatility of central schemes as black-box, Jacobian-free MHD solvers.