MATH 255 **Applied Honors Calculus III** **Winter 2005**

Review Sheet for 2nd Midterm Exam – Part II

1. True or False. Determine whether the following statements are true or false and provide an explanation for your answer (5 points each).

(a) If a particle's trayectory is described by the vector curve $\mathbf{r}(t)$ satisfying $|\mathbf{r}'(t)| = c$, for all values of t, where c is a constant, then its acceletation vector satisfies, $\mathbf{a}(t) = \mathbf{0}$.

(b) If the position vector $\mathbf{r}(t)$ of a moving particle satisfies $|\mathbf{r}(t)| = c$ for all values of t, where c is constant, then the particle path is a straight line.

(c) If a function $f(x, y)$ satisfies $f(0, 0) = \frac{1}{2}$ and

$$\lim_{(x,y)\to(0,0)} f(x, y) = \frac{1}{2}$$

along the paths $L_1 : y = x$ and $L_2 : y = x^2$, then the function is continuous at $(0, 0)$.

(d) The function $f(x, y) = \frac{\cos x}{x^2+y^2+2}$ is continuous everywhere in $\mathbb{R}^2$.

(e) The function $f(x, y) = \frac{x^4+2y^2+3}{x^2+y^2-1}$ is continuous everywhere in $\mathbb{R}^2$.

(f) The function

$$f(x, y) = \begin{cases} \frac{x^2+\sin^2 y}{x^2+2y^2} & \text{if} \quad (x, y) \neq (0, 0) \\ \\ 0 & \text{if} \quad (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$.

(g)

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy\, dx = 4\pi$$

(h)

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4 - x^2 - y^2}\, dy\, dx = \frac{32}{3}\pi$$

2. (15 points) Consider the solid bounded by the elliptic paraboloid $z = 2x^2 + y^2$ and the plane $z = 4$. (a) Sketch the solid. Find (b) its volume, and (c) its surface area.

3. (10 points) Consider the region D of the xy-plane bounded by the curves $x = 2$, $y = 0$, and $y = x^2$. (a) Sketch D, and express the double integral

$$\iint_D f(x, y)\, dA$$

as an iterated integral of the form: (before you proceed with (b) and (c), read the note on the back)

(b) $\int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x, y)\, dy\, dx$

(c) $\int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x,y)\ dx\ dy$

CAUTION: the limits of integration x_1, x_2, y_1, and y_2 may or may not be constants.

4. (10 points) Find the center of mass of a thin plate whose shape is the region of the xy-plane bounded by the curves $y = 2 - \sqrt{4 - x^2}$ and $y = \sqrt{4 - (x-2)^2}$ and has density $\rho(x,y) = k(x^2 + y^2)$

5. (15 points) Evaluate the integrals:

(a)

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} dz\ dy\ dx.$$

(b)

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} (x^2+y^2)^{3/2}\ dz\ dy\ dx.$$