

EXTRA EXERCISES FOR CHAPTER 13, GROUP 3

H13.16 - 13.24: If the sentence is FO-valid, open **Proof CStern 13x**, and complete and save the proof. If it is FO-invalid, use Tarski's World to give a counterexample. (Start by opening **Sentences CStern 13x**. Save your solution as **World CStern 13x**.)

H 13.16 $\exists x \neg \text{Dodec}(x) \rightarrow \exists x (\text{Dodec}(x) \rightarrow \text{Small}(x))$

H 13.17 $\neg \forall x \text{Large}(x) \vee \forall x (\text{Medium}(x) \rightarrow \text{Large}(x))$

H 13.18 $\forall x (\text{Tet}(x) \vee \text{Cube}(x)) \rightarrow (\exists x \text{Tet}(x) \vee \forall x \text{Cube}(x))$

H 13.19 $\forall x (\text{Cube}(x) \rightarrow \text{Small}(x)) \rightarrow (\exists x \text{Cube}(x) \rightarrow \exists x \text{Small}(x))$

H 13.20 $\forall x (\text{Tet}(x) \rightarrow \neg \text{Medium}(x)) \vee \exists x (\text{Tet}(x) \wedge \text{Medium}(x))$

H 13.21

$\exists x \exists y (\text{Cube}(x) \wedge \text{Tet}(y) \wedge \neg \text{SameCol}(x,y)) \rightarrow \neg \exists x \exists y (\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameCol}(x,y))$

H 13.22 $\forall x \exists y (\text{Small}(x) \rightarrow \text{RightOf}(x,y)) \rightarrow (\forall x \neg \text{Small}(x) \vee \exists x \exists y \text{RightOf}(x,y))$

H 13.23 $\exists x \forall y \neg \text{Larger}(x,y) \rightarrow \forall y \exists x \neg \text{Larger}(x,y)$

H 13.24

$\exists x [\text{Cube}(x) \wedge \neg \forall y (\text{Tet}(y) \rightarrow \text{Larger}(y,x))] \rightarrow \exists x [\text{Cube}(x) \wedge \forall y (\text{Tet}(y) \rightarrow \neg \text{Larger}(y,x))]$

H 13.25 - H 13.32: Open **Proof CStern 13x** for H 13.x. Use Fitch to give a proof showing each pair of sentences is FO-equivalent.

H 13.25 $\exists x (\text{LeftOf}(x,a) \vee \text{BackOf}(x,a)), \exists x \text{LeftOf}(x,a) \vee \exists x \text{BackOf}(x,a)$

H 13.26 $\forall x (\text{Small}(x) \wedge \text{Tet}(x)), \forall x \text{Small}(x) \wedge \forall x \text{Tet}(x)$

H 13.27 $\exists x (\text{Dodec}(x) \leftrightarrow \neg \text{Large}(x)), \neg \forall x (\text{Dodec}(x) \leftrightarrow \text{Large}(x))$

H 13.28 $\forall x (\text{FrontOf}(a,x) \rightarrow \neg \text{Large}(x)), \neg \exists x (\text{FrontOf}(a,x) \wedge \text{Large}(x))$

H 13.29 $\forall x (\text{Tet}(x) \rightarrow \text{Medium}(a)), \neg \exists x \text{Tet}(x) \vee \text{Medium}(a)$

H 13.30 $\exists x \text{Large}(x) \vee \text{Medium}(b), \forall x \neg \text{Large}(x) \rightarrow \text{Medium}(b)$

H 13.31 $\exists x \text{Dodec}(x) \rightarrow \text{Small}(a), \forall x (\text{Dodec}(x) \rightarrow \text{Small}(a))$

H 13.32 $\forall x \exists y (S(x) \rightarrow R(x,y)), \forall x (S(x) \rightarrow \exists y R(x,y))$

H 13.33 - H 13.34 Open **Sentences CStern 13x**. To show each pair of sentences is not FO-equivalent build a world where the first is true but the second is false. Save as **World CStern 13x**.

H 13.33 1. $\forall x \forall y (\text{SameCol}(x,y) \rightarrow \text{SameSize}(x,y))$

2. $\forall x \forall y (\text{SameSize}(x,y) \rightarrow \text{SameCol}(x,y))$

H 13.34 1. $\forall x \exists y \neg \text{Larger}(x,y)$

2. $\exists y \forall x \neg \text{Larger}(x,y)$

H 13.35 - 13.38 **Open Proof C**Stern 13x. Use Fitch to give a proof showing that each set of sentences is FO-inconsistent.

H 13.35 $\{\forall x (J(a,x) \leftrightarrow (K(x) \vee L(x)), \exists x (J(a,x) \wedge \neg K(x)), \neg \exists x L(x)\}$

H 13.36 $\{\exists x \text{ Cube}(x), \exists x \forall y \text{ SameCol}(x,y), \forall x (\exists y \text{ SameCol}(y,x) \rightarrow \neg \text{Cube}(x))\}$

H 13.37 $\{\forall x (\exists y \neg \text{SameShape}(x,y) \rightarrow \forall y \text{SameSize}(x,y)),$
 $\neg \exists x \exists y (\text{SameSize}(x,y) \wedge \text{Adjoins}(x,y)),$
 $\forall x \forall y (\text{SameShape}(x,y) \vee \text{Adjoins}(x,y)),$
 $\exists x \exists y \neg (\text{SameShape}(x,y) \vee \text{Dodec}(x))\}$

H 13.38 $\{\forall x (B(x) \rightarrow \exists y (C(x,y))),$
 $\forall x \forall y (C(x,y) \leftrightarrow \neg D(y,x)),$
 $\forall x \forall y (D(x,y) \vee G(x,y)),$
 $\exists x (B(x) \wedge \neg \exists y G(y,x))\}$

H 13.39 - H 13.41 Use Tarski's World to show that the set with the listed sentences as its members is FO-consistent:

H 13.39 1. $\neg \exists x (\text{Tet}(x) \wedge \forall y (\text{Cube}(y) \rightarrow \text{Adjoins}(x,y)))$
 2. $\forall y (\text{Cube}(y) \rightarrow \exists x (\text{Tet}(x) \wedge \text{Adjoins}(x,y)))$
 3. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Tet}(y)) \rightarrow \text{SameSize}(x,y))$
 4. $\exists x \exists y (\text{Dodec}(x) \wedge \text{Dodec}(y) \wedge x \neq y \wedge \text{SameSize}(x,y))$
 5. $\neg \forall x \forall y ((\text{Dodec}(x) \wedge \text{Dodec}(y)) \rightarrow \text{SameSize}(x,y))$
 6. $\exists x \exists y (\text{Cube}(x) \wedge \text{Cube}(y) \wedge \neg \text{SameSize}(x,y))$

H 13.40 1. $\forall x \forall y ((\text{Cube}(x) \wedge \text{Dodec}(y)) \rightarrow \text{BackOf}(x,y))$
 2. $\exists x \exists y \exists z (\text{Dodec}(x) \wedge \text{Tet}(y) \wedge \text{Tet}(z) \wedge \text{Between}(x,y,z))$
 3. $\exists x (\text{Cube}(x) \wedge \exists y (\text{Tet}(y) \wedge \text{SameRow}(x,y)))$
 4. $\neg \forall x \forall y ((\text{Cube}(x) \wedge \text{Cube}(y)) \rightarrow \text{SameRow}(x,y))$
 5. $\neg \exists x \exists y (\text{Tet}(x) \wedge \text{Tet}(y) \wedge x \neq y \wedge \text{SameRow}(x,y))$
 6. $\exists x \exists y (\text{Dodec}(x) \wedge \text{Dodec}(y) \wedge \neg (\text{SameRow}(x,y) \vee \text{SameCol}(x,y)))$

H 13.41 1. $\text{Cube}(a) \wedge \text{Tet}(b) \wedge \text{Tet}(c) \wedge b \neq c$
 2. $\neg \forall x (\text{Tet}(x) \rightarrow (x=b \vee x=c))$
 3. $\forall x (\text{Tet}(x) \rightarrow \exists y (\text{Dodec}(y) \wedge \text{Adjoins}(y,x)))$
 4. $\neg \exists x \exists y (\text{Cube}(x) \wedge \text{Dodec}(y) \wedge \text{SameCol}(x,y))$
 5. $\exists x \exists y (\text{Cube}(x) \wedge \text{Cube}(y) \wedge x \neq y \wedge \neg \exists z [\text{Cube}(z) \wedge z \neq x \wedge z \neq y])$
 6. $\forall x \forall y [\neg (\text{Tet}(x) \wedge \text{Tet}(y)) \vee \text{SameSize}(x,y)]$
 7. $\forall x \forall y [(\text{Cube}(x) \wedge \text{Tet}(y)) \rightarrow \text{Larger}(x,y)]$
 8. $\forall x [\text{Cube}(x) \rightarrow \exists y \exists z (\text{Dodec}(y) \wedge \text{Dodec}(z) \wedge \text{Between}(x,y,z))]$
 9. $\forall x \forall y [(\text{Cube}(x) \wedge \text{Cube}(y) \wedge x \neq y) \rightarrow \neg \text{SameSize}(x,y)]$
 10. $\exists x (\text{Dodec}(x) \wedge \forall y [(\text{Dodec}(y) \wedge y \neq x) \rightarrow \text{Larger}(x,y)])$