Truth-functional compounds containing quantifiers

Domain limited to people in this room
1. If anyone cheats, someone will be punished.
2. If anyone cheats, he or she will be punished.
3. Only those who cheat will be punished.
4. If everyone studies, no one will fail.
5. No one who studies will fail.
   (Anyone who studies will not fail.)
6. I'll be disappointed in anyone who cheats.
7. If anyone cheats, I'll be disappointed.
8. Everyone will be punished unless someone confesses.

Symbolization with multiple quantifiers
Domain = people, \( L(x,y) = x \) likes \( y \)
\( \forall x \forall y L(x,y) = \) There is someone who likes everyone.
\( \exists x \forall y L(x,y) = \) There is someone whom everyone likes.
\( \forall x \exists y L(x,y) = \) Everyone likes someone (or other).
\( \exists x \exists y L(x,y) = \) Everyone is liked by someone (or other).
   (For everyone, there is someone who likes that person.)

Domain not limited: 
\( F(x,y) = x \) is a flavor of \( y \)  
\( i = \) ice cream  
\( P(x) = x \) is a person  
\( L(x,y) = x \) likes \( y \)
1. Everyone likes all flavors of ice cream.
2. Some people like all flavors of ice cream.
3. Nobody likes every flavor of ice cream.
4. For every flavor of ice cream, there is someone who likes it.
5. Everyone likes at least one flavor of ice cream.
6. There's a flavor of ice cream everyone likes.
Identity and numbers

Dictionary:  Domain = states in the US  a = Alaska  c = California
L(x,y) = x is larger than y  t = Texas

1. Alaska is larger than any other state.
2. Alaska is the only state that is larger than Texas.
3. No state is larger than Alaska.
4. At least one state is larger than Texas.
5. At least two states are larger than California.
6. At most one state is larger than Texas.
7. At most two states are larger than California.
8. Exactly one state is larger than Texas.
9. Exactly two states are larger than California.
10. Alaska and Texas are the only states that are larger than California.

Dictionary:

\begin{align*}
f &= \text{Felix} & C(x) &= x \text{ is a cat} \\
g &= \text{Garfield} & P(x) &= x \text{ is a person} \\
t &= \text{Tina} & F(x,y) &= x \text{ is more famous than } y \\
\end{align*}

\begin{align*}
h &= \text{Herman} & H(x,y) &= x \text{ hates } y \\
j &= \text{Jason} & L(x,y) &= x \text{ likes } y \\
\end{align*}

Domain = people

\begin{align*}
1. \text{ Everyone likes at least one cat.} & & 13. \text{ No one hates himself.} \\
2. \text{ There's no cat that everyone likes.} & & 14. \text{ Everyone likes himself.} \\
3. \text{ Felix is the only cat Tina likes.} & & 15. \text{ Nobody hates everybody.} \\
4. \text{ There is only one cat that Tina likes.} & & 16. \text{ Nobody hates anybody.} \\
5. \text{ Tina likes at most one cat.} & & 17. \text{ Nobody hates anybody who likes him or her.} \\
6. \text{ Tina likes at least two cats.} & & 18. \text{ Everyone who likes anyone likes himself or herself.} \\
7. \text{ Tina likes exactly two cats.} & & 19. \text{ If Jason likes anyone, he likes Herman.} \\
8. \text{ Tina likes at most two cats.} & & 20. \text{ Herman hates everyone except Jason.} \\
9. \text{ Tina likes at least three cats.} & & 21. \text{ Jason and Herman do not like any of the same people.} \\
10. \text{ If any cat is more famous than Felix, Garfield is.} & & 22. \text{ There is at least one person who likes, and is liked by, Jason.} \\
11. \text{ Felix is more famous than any other cat.} & & 23. \text{ There are exactly two people whom both Jason and Herman like.} \\
12. \text{ Felix is more famous than any other cat except Garfield.} & & 24. \text{ Jason hates at most three people.} \\
\end{align*}
Answers to Exercises

1. \( \forall x(P(x) \rightarrow \exists y[C(y) \land L(x,y)]) \)
2. \( \neg \exists x(C(x) \land \exists y(P(y) \land L(y,x))) \)
   \( \exists x(C(x) \rightarrow \neg \forall y[P(y) \land H(y,x)]) \)
3. \( \forall (f) \land \forall (L(f)) \land \forall x[(C(x) \land L(t,x)) \rightarrow x=f] \)
   \( \forall (f) \land \forall (L(f)) \land \forall x[(C(x) \land L(t,x)) \neg x=f] \)
4. \( \exists x(C(x) \land L(t,x)) \land \exists y[(C(y) \land L(t,y)) \rightarrow y=x]) \)
   \( \exists x(C(x) \land L(t,x)) \land \neg \exists y[(C(y) \land L(t,y)) \land y=x]) \)
5. \( \neg \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y))) \land x \neq y] \)
6. \( \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y))) \land x \neq y] \)

7. \( \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y))) \land x \neq y] \)
   \( \neg (z(C(z) \land L(t,z)) \rightarrow (z \neq x \land z \neq y)) \)
   \( \neg (z(C(z) \land L(t,z)) \rightarrow (z \neq x \land z \neq y)) \)
   \( \neg (z(C(z) \land L(t,z)) \rightarrow (z \neq x \land z \neq y)) \)
   \( \neg (z(C(z) \land L(t,z)) \rightarrow (z \neq x \land z \neq y)) \)

8. \( \neg \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y)) \land L(t,z)) \land \exists x \neg x \neq z \land y \neq z] \)
   \( \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y)) \land L(t,z)) \land \exists x \neg x \neq z \land y \neq z] \)

9. \( \exists x[y((C(x) \land C(y)) \land (L(t,x) \land L(t,y)) \land L(t,z)) \land \exists x \neg x \neq z \land y \neq z] \)

10. \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
    \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
11. \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
    \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
12. \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
    \( \exists x[(C(x) \land \neg F(f,x)) \land F(g,f)] \)
13. \( \exists x[H(x,y)] \)
    \( \exists x[H(x,y)] \)
14. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
15. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
16. \( \exists x[yL(x,y)] \)
    \( \exists x[yL(x,y)] \)
17. \( \exists x[yL(x,y)] \)
    \( \exists x[yL(x,y)] \)
18. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
19. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
20. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
21. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
22. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
23. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)
24. \( \exists x[L(x,y)] \)
    \( \exists x[L(x,y)] \)