Truth-functional compounds containing quantifiers

Domain limited to people in this room

1. If anyone cheats, someone will be punished.

2. If anyone cheats, he or she will be punished.

3. Only those who cheat will be punished.

4. If everyone studies, no one will fail.

5. No one who studies will fail.

   (Anyone who studies will not fail.)

6. I'll be disappointed in anyone who cheats.

7. If anyone cheats, I'll be disappointed.

8. Everyone will be punished unless someone confesses.

Symbolization with multiple quantifiers

Domain = people, $L(x,y)$ = x likes y

$\exists x \forall y L(x,y)$ = There is someone who likes everyone.
$\exists x \forall y L(y,x)$ = There is someone whom everyone likes.
$\forall x \exists y L(x,y)$ = Everyone likes someone (or other).
$\forall x \exists y L(y,x)$ = Everyone is liked by someone (or other).
(For everyone, there is someone who likes that person.)

Domain not limited: $F(x,y)$ = x is a flavor of y; $P(x)$ = x is a person; i = ice cream; $L(x,y)$= x likes y

1. Everyone likes all flavors of ice cream.

2. Some people like all flavors of ice cream.

3. Nobody likes every flavor of ice cream.

4. For every flavor of ice cream, there is someone who likes it.

5. Everyone likes at least one flavor of ice cream.

6. There's a flavor of ice cream everyone likes.

Identity and numbers Dictionary: Domain = states in the US; a = Alaska; c = California; L(x,y) = x is larger than y; t = Texas

1. Alaska is larger than any other state.
2. Alaska is the only state that is larger than Texas.
3. No state is larger than Alaska.
4. At least one state is larger than Texas.
5. At least two states are larger than California.
6. At most one state is larger than Texas.
7. At most two states are larger than California.
8. Exactly one state is larger than Texas.
9. Exactly two states are larger than California.
10. Alaska and Texas are the only states that are larger than California.

Dictionary:
f = Felix C(x) = x is a cat
g = Garfield P(x) = x is a person
t = Tina F(x,y) = x is more famous than y
L(x,y) = x likes y

1. Everyone likes at least one cat.
2. There's no cat that everyone likes.
3. Felix is the only cat Tina likes.
4. There is only one cat that Tina likes.
5. Tina likes at most one cat.
6. Tina likes at least two cats.
7. Tina likes exactly two cats.
8. Tina likes at most two cats.
9. Tina likes at least three cats.
10. If any cat is more famous than Felix, Garfield is.
11. Felix is more famous than any other cat.
12. Felix is more famous than any other cat except Garfield.

Dictionary:
h = Herman H(x,y) = x hates y
j = Jason L(x,y) = x likes y
Domain = people

13. No one hates himself.
14. Everyone likes himself.
15. Nobody hates everybody.
16. Nobody hates anybody.
17. Nobody hates anybody who likes him or her.
18. Everyone who likes anyone likes himself or herself.
19. If Jason likes anyone, he likes Herman.
20. Herman hates everyone except Jason.
21. Jason and Herman do not like any of the same people.
22. There is at least one person who likes, and is liked by, Jason.
23. There are exactly two people whom both Jason and Herman like.
24. Jason hates at most three people.

Answers to Exercises

1. □x(P(x) → □y[C(y) □ L(x,y)])
2. ¬□x(C(x) □ □y[P(y) □ L(y,x)])
   □x(C(x) → □y[P(y) □ ¬H(y,x)])
3. (C(f) □ L(t,f)) □ □x([C(x) □ L(t,x)] → x=f)
   (C(f) □ L(t,f)) □ ¬□x([C(x) □ L(t,x)] □ x≠f)
4. □x([C(x) □ L(t,x)] □ □y([C(y) □ L(t,y)] → y=x))
   □x([C(x) □ L(t,x)] □ ¬□y([C(y) □ L(t,y)) □ y≠x))
5. ¬□x□y([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y)
6. □x□y([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y)
7. □x□y[([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y) □ ¬□z([C(z) □ L(t,z)] □ (z≠x □ z≠y))]
   □x□y[([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y) □ □z([C(z) □ L(t,z)] → (z=x v z=y))]
   □x□y[([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y) □ □z[¬(z=x v z=y) → ¬(C(z) □ L(t,z))]]
   □x□y[([(C(x) □ C(y)) □ (L(t,x) □ L(t,y))] □ x≠y) □ □z[(z≠x □ z≠y) → ¬(C(z) □ L(t,z))]]
8. ¬□x□y□z[([(C(x) □ C(y)) □ C(z)] □ [(L(t,x) □ L(t,y)) □ L(t,z)]) □ [(x≠y □ x≠z) □ y≠z]]
9. □x□y□z[([(C(x) □ C(y))□ C(z)] □ [(L(t,x) □ L(t,y)) □ L(t,z)]) □ [(x≠y □ x≠z) □ y≠z]]
10. □x[C(x) □ F(x,f)] □ F(g,f)
    □x([C(x) □ F(x,f)] □ F(g,f))
11. C(f) □ □x[(C(x) □ x≠f) □ F(f,x)]
    C(f) □ □x[C(x) □ (x≠f □ F(f,x))]
12. [(C(f) □ C(g)) □ ¬F(f,g)] □ □x[(C(x) □ (x≠f □ x≠g)) □ F(f,x)]
    [(C(f) □ C(g)) □ ¬F(f,g)] □ □x[(C(x) □ ¬(x=f v x=g)) □ F(f,x)]
    [(C(f) □ C(g)) □ ¬F(f,g)] □ □x[(C(x) □ ¬F(f,x)) □ (x=f v x=g)]
    [(C(f) □ C(g)) □ ¬F(f,g)] □ ¬□x[(C(x) □ (x≠f □ x≠g)) □ ¬F(f,x)]
13. ¬□xH(x,x)   (preferred)
    □x¬H(x,x)
14. □xL(x,x)
15. ¬□x□yH(x,y)   (preferred)
    □x□y¬H(x,y)
16. ¬□x□yH(x,y)   (preferred)
    □x□y¬H(x,y)
17. ¬□x□y(L(y,x) □ (x,y))   or   ¬□x□y(L(x,y) □ H(y,x))
    □x□y(L(y,x) □ ¬H(x,y))   or   □x□y(L(x,y) □ ¬H(y,x))
18. □x(□yL(x,y) □ L(x,x))   (preferred)
    □x(¬L(x,x) □ ¬□□□yL(x,y))
19. □xL(j,x) □ L(j,h)   (preferred)
    □x[L(j,x) □ L(j,h)]
20. ¬H(h,j) □ □x(x≠j □ H(h,x))   (preferred)
    ¬H(h,j) □ □x(x=j   H(h,x))
    ¬H(h,j) □ ¬□x(x≠j □ ¬H(h,x))
21. ¬□x[L(j,x) □ L(h,x)]   (preferred)
    □x¬[L(j,x) □ L(h,x)]
    □x[¬L(j,x)   ¬L(h,x)]
22. □x(L(x,j) □ L(j,x))   (preferred)
    ¬□x(L(x,j) □ ¬L(j,x))
23. □x□y[([(L(j,x) □ L(h,x)) □ (L(j,y) □ L(h,y))] □ x≠y) □ ¬□z[(L(j,z) □ L(h,z)) □ (z≠x □ z≠y)]]
    □x□y[([(L(j,x) □ L(h,x)) □ (L(j,y) □ L(h,y))] □ x≠y) □ □z([L(j,z) □ L(h,z)] □ (z=x   z=y))]
    □x□y[([(L(j,x) □ L(h,x)) □ (L(j,y) □ L(h,y))] □ x≠y) □ □z[¬(z=x   z=y) □ ¬(L(j,z) □ L(h,z))]]
    □x□y[([(L(j,x) □ L(h,x)) □ (L(j,y) □ L(h,y))] □ x≠y) □ □z[(z≠x □ z≠y) □ ¬(L(j,z) □ L(h,z))]]
24. ¬□x_1□x_2□x_3□x_4([([H(j,x_1) □ H(j,x_2)] □ H(j,x_3)) □ H(j,x_4)] □ ([(x_1≠x_2 □ x_1≠x_3) □ x_1≠x_4] □ (x_2≠x_3 □ x_2≠x_4) □ x_3≠x_4)]