Proof Rules

= Intro

```
   | …
 i | a = a        =Intro
```

= Elim

```
 i | P(a)
   | …
 k | a = b
   | …
   | P(b)         =Elim,: i,k
```

∧ Intro

```
   | …
 i | A
   | …
 k | B
   | …
   | A ∧ B    ∧Intro: i,k
```

∧ Elim

```
   | …
 i | A ∧ B
   | …
   | A      ∧Elim: i
```

OR:

```
   | …
 i | A ∧ B
   | …
   | B      ∧Elim,: i
```

∨ Intro

```
   | …
 i | A
   | …
   | A ∨ B     ∨Intro, i
```

OR:

```
   | …
 i | A
   | …
   | B ∨ A    ∨Intro: i
```

∨ Elim

```
   | …
 i | A ∨ B
   | …
 k | | A
   | |  …
 m | | C
   | …
 n | | B
   | |  …
 p | | C
   | C   ∨Elim: i,k-m,n-p
```

⊥ Intro

```
   | …
 i | A
   | …
 k | ¬A
   | …
   | ⊥       ⊥Intro: i,k
```

⊥ Elim

```
   | …
 i | ⊥
   | …
   | C   ⊥Elim: i
```

¬ Intro

```
   | …
 i | | A
   | |  …
 k | | ⊥
   | ¬A    ¬Intro: i -k
```

¬ Elim

```
   | …
 i | ¬¬A
   | …
   | A      ¬Elim: i
```

→ Intro

```
   | …
 i | | A
   | |  …
 k | | B
   | A→B  →Intro:i -k
```

→ Elim

```
   | …
 i | A → B
   | …
 k | A
   | …
   | B   →Elim:i,k
```

↔ Intro

```
   | …
 i | | A
   | |  …
 k | | B
   |
 l | | B
   | |  …
 n | | A
   | A↔B   ↔Intro:i-k,l-n
```

↔ Elim

```
   | …
 i | A ↔ B
   | …
 k | A
   | …
   | B   ↔Elim: i,k
```

OR:

```
   | …
 i | A ↔ B
   | …
 k | B
   | …
   | A   ↔Elim: i,k
```

PROOFS: Some very easy problems to start on

Problems 0.1, 0.1, and 8.1 - 8.14 do not require subderivations. For 8.15 - 8.23, subderivations may be needed. Use Fitch to open Proof CStern 080x for H 8.1 - 8.9, and Proof CStern 08xx for H 8.10 -8.24. Submit assigned solutions to the GradeGrinder. Do not use TautCon. Apply AnaCon only to literals (atomic sentences and their negations).

H 0.1 The starter must be malfunctioning. The car won't start, but the lights are working. If the lights work, the battery must be charged. If the car won't start when the battery is charged, there must be a problem with the starter. (S = The starter is functioning properly; C = The car starts; L = The lights are working; B = The battery is charged)

H 0.2 If Mia moved out of her parents' house, she must be able to afford to. She can afford it only if she found both a roommate and a job. Since she moved out, she must have found a job.

H 8.1 $\{\mathrm{Dodec}(a), \mathrm{Dodec}(a) \rightarrow \mathrm{Medium}(a)\}\ /\therefore\ \mathrm{Dodec}(a) \wedge \mathrm{Medium}(a)$

H 8.2 $\{\mathrm{Tet}(a), \mathrm{Tet}(a) \rightarrow \mathrm{Tet}(b), (\mathrm{Tet}(a) \wedge \mathrm{Tet}(b)) \rightarrow \mathrm{Larger}(a,b)\}/\therefore\ \mathrm{Tet}(b) \wedge \neg\mathrm{SameSize}(a,b)$

H 8.3 $\{\mathrm{SameCol}(b,c) \rightarrow (\mathrm{Dodec}(b) \wedge \mathrm{Cube}(c)), \mathrm{SameCol}(b,c), \mathrm{Cube}(c) \rightarrow c{=}d\}\ /\therefore\ \mathrm{Cube}(d)$

H 8.6
$\mathrm{Small}(a)$
$\mathrm{Cube}(a) \rightarrow (\mathrm{Large}(c) \rightarrow \mathrm{Cube}(c))$
$(\mathrm{Small}(a) \wedge \mathrm{Medium}(b)) \rightarrow \mathrm{Large}(c))$
$\underline{\mathrm{Cube}(a) \wedge \mathrm{Medium}(b))}$
$\mathrm{SameShape}(a,c)$

H 8.7
$(\mathrm{Tet}(b) \vee \mathrm{Dodec}(b)) \rightarrow \mathrm{Small}(b)$
$\mathrm{Tet}(b) \leftrightarrow \mathrm{Medium}(c)$
$\underline{\mathrm{Medium}(c)}$
$\mathrm{Larger}(c,b)$

H 8.8 $\{ (K \wedge L) \leftrightarrow (B \wedge C), C \wedge L, B\}\ /\therefore\ ((B \wedge C) \wedge K) \wedge L$

H 8.9 $\{ [(A \wedge B) \wedge C] \rightarrow D,\ A \rightarrow (R \rightarrow C), A \wedge R, (C \wedge A) \rightarrow B\}\ /\therefore\ D$

H 8.10 $\{ (J \vee S) \leftrightarrow L, S \}\ /\therefore\ L$

H 8.11 $\{ A, (C \vee A) \rightarrow (T \wedge M) \}\ /\therefore\ T \wedge M$

H 8.12 $\{ B \wedge E, (E \vee (F \wedge C)) \rightarrow K \}\ /\therefore\ M \vee K$

H 8.13 $\{ (Q \wedge M) \leftrightarrow (P \vee R),\ P \}\ /\therefore\ M$

H 8.14 $\{(G \vee B) \rightarrow L,\ (S \wedge L) \rightarrow H, S \wedge B\}\ /\therefore\ L \wedge H$

H 8.15. $\{P, \neg(P \wedge Q)\}\ /\therefore\ \neg Q$

H 8.16
$\mathrm{Cube}(a) \rightarrow \mathrm{Tet}(b)$
$\underline{\mathrm{Tet}(b) \rightarrow \mathrm{Dodec}(c)}$
$\mathrm{Cube}(a) \rightarrow \mathrm{Dodec}(c)$

H 8.17
$\neg\mathrm{Small}(b)$
$\underline{b{=}c \rightarrow \mathrm{Small}(b)}$
$\mathrm{Small}(b) \leftrightarrow b{=}c$

H 8.18
$\mathrm{Tet}(a) \vee \mathrm{Tet}(b)$
$\underline{\neg(\mathrm{Tet}\ (a) \wedge \mathrm{Tet}(b))}$
$\neg\mathrm{Tet}\ (a) \leftrightarrow \mathrm{Tet}\ (b)$

H 8.19
$\mathrm{RightOf}(a,b) \vee \mathrm{SameRow}(a,b)$
$\mathrm{RightOf}(a,b) \rightarrow (\mathrm{BackOf}(a,b) \wedge \mathrm{Larger}(a,b))$
$\underline{\mathrm{SameRow}(a,b) \leftrightarrow \mathrm{Larger}(a,b)}$
$\mathrm{Larger}(a,b)$ $\mathrm{Cube}(c)$

H 8.20
$\mathrm{Cube}(a) \vee \mathrm{Cube}(b) \vee \mathrm{Cube}(c)$
$\mathrm{Cube}(a) \rightarrow \mathrm{Cube}(b)$
$\underline{\mathrm{Cube}(c) \leftrightarrow \mathrm{Cube}(b)}$
$\mathrm{Cube}(c)$

H 8.21 $\{ (J \wedge K) \rightarrow L, J \wedge M \}\ /\therefore\ \neg L \rightarrow \neg K$

H 8.22 $\{ A \rightarrow B,\ (A \wedge B) \rightarrow C, (C \vee D) \rightarrow A \}\ /\therefore\ A \leftrightarrow C$

H 8.23 $\{ L \rightarrow (F \vee G),\ F \rightarrow (J \leftrightarrow K), K \wedge L, (G \wedge L) \rightarrow M \}\ /\therefore\ J \vee M$

H 8.24 $\{ (P \wedge Q) \leftrightarrow (\neg R \vee \neg S) \}\ /\therefore\ (P \wedge S) \rightarrow (Q \rightarrow \neg R)$

SAMPLE ANSWERS TO SELECTED PRACTICE PROBLEMS

H 0.1

$\{\neg C \wedge L, L \rightarrow B, (\neg C \wedge B) \rightarrow \neg S\}$ /∴ $\neg S$

1	$\neg C \wedge L$	P
2	$L \rightarrow B$	P
3	$\underline{(\neg C \wedge B) \rightarrow \neg S}$	P
4	$\neg C$	∧Elim: 1
5	L	∧Elim: 1
6	B	→Elim: 2,5
7	$\neg C \wedge B$	∧Intro: 4,6
8	$\neg S$	→Elim: 7,3

2. $\{ M \rightarrow A, A \rightarrow (R \wedge J), M \}$ /∴ J
(The second premise could be symbolized instead as
$\neg (R \wedge J) \rightarrow \neg A$
The derivation would then require a subderivation.)

1	$M \rightarrow A$	P
2	$A \rightarrow (R \wedge J)$	P
3	$\underline{M}$	P
4	A	→ Elim: 1,3
5	$R \wedge J$	→Elim: 2,4
6	J	∧Elim: 5

H 8.11

1	A	P
2	$\underline{(C \vee A) \rightarrow (T \wedge M)}$	P
3	$C \vee A$	∨Intro: 1
4	$T \wedge M$	→Elim: 2,3

H 8.12

1	$B \wedge E$	P
2	$\underline{[E \vee (F \wedge C)] \rightarrow K}$	P
3	E	∧Elim: 1
4	$E \vee (F \wedge C)$	∨Intro: 3
5	K	→Elim: 2,4
6	$M \vee K$	∨Intro: 5

H 8.13

1	$(Q \wedge M) \leftrightarrow (P \vee R)$	P
2	$\underline{P}$	P
3	$P \vee R$	∨Intro: 2
4	$Q \wedge M$	↔Elim: 1,3
5	M	∧Elim: 4

H 8.21

1	$(J \wedge K) \rightarrow L$	P
2	$\underline{J \wedge M}$	P
3	$\mid \underline{\neg L}$	
4	$\mid \mid \underline{K}$	
5	$\mid \mid J$	∧Elim: 2
6	$\mid \mid J \wedge K$	∧Intro:4,5
7	$\mid \mid L$	→Elim: 6,1
8	$\mid \mid \bot$	⊥Intro: 3,7
9	$\mid \neg K$	¬Intro: 4-(7,8)
10	$\neg L \rightarrow \neg K$	→Intro: 3-9

H 8.22

1	$A \rightarrow B$	P
2	$(A \wedge B) \leftrightarrow C$	P
3	$\underline{(C \vee D) \rightarrow A}$	P
4	$\mid \underline{A}$	
5	$\mid B$	→Elim: 1,4
6	$\mid A \wedge B$	∧Intro: 4,5
7	$\mid C$	↔Elim: 2,6
8	$\mid \underline{C}$	
9	$\mid C \vee D$	∨Intro: 8
10	$\mid A$	→Elim: 3.9
11	$A \leftrightarrow C$	↔Intro: 4-7,8-10

H 8.23

1	$L \rightarrow (F \vee G)$	P
2	$F \rightarrow (J \leftrightarrow K)$	P
3	$K \wedge L$	P
4	$\underline{(G \wedge L) \rightarrow M}$	P
5	L	∧Elim: 3
6	$F \vee G$	→Elim: 1,5
7	$\mid \underline{F}$	
8	$\mid J \leftrightarrow K$	→Elim: 2,7
9	$\mid K$	∧Elim: 3
10	$\mid J$	↔Elim: 8,9
11	$\mid J \vee M$	∨Intro: 10
12	$\mid \underline{G}$	
13	$\mid G \wedge L$	∧Intro: 12,5
14	$\mid M$	→Elim: 4,13
15	$\mid J \vee M$	∨Intro: 14
16	$J \vee M$	∨Elim: 6,7-11,12-15

STRATEGIES FOR PROOFS

1. Try to pull goal sentence out of a sentence you already have that contains the goal sentence as a component. For example, suppose your goal is '(J ∧ K)' and one of your earlier sentences (either a premise or something you have already derived) is '((J ∧ K) ↔ L)'. In that case, aim for 'L', and then use it with '((J ∧ K) ↔ L)' to get '(J ∧ K)' by ↔Elim.

2. If goal sentence cannot be extracted as a whole from any sentence you already have, base your strategy on the structure of the goal sentence.

∧	conjunction	Aim for each conjunct separately, then apply ∧Intro.
→	conditional	Plan to use →Intro. To do this, start a subderivation with the antecedent as provisional assumption. Aim for the consequent in the subderivation.
↔	biconditional	Plan to use ↔Intro. Start one subderivation with the left side of the biconditional and aim for the right in this subderivation. Set up a second subderivation going from the right side of the biconditional to the left.
¬	negation	If goal has ¬ as its main operator, try reaching it by ¬Intro. To do this, start a subderivation with the statement to be negated (but without the ¬) as provisional assumption. Within this subderivation, get ⊥ by aiming for any contradiction you can get.
∨	disjunction	a) If one disjunct is obviously easy to get, get that one. Then use ∨Intro to reach goal. b) If neither disjunct is obviously easy to get, look for an earlier disjunction. Try ∨Elim on earlier disjunction c) If neither of these works, assume the negation of the goal sentence. You will need to use ⊥Intro followed by ¬Intro and then ¬¬Elim to reach the goal.

3. If you have a disjunction already, and you can't use it with one of our easy rules, you will probably have to use the ∨Elim rule. If you will have to use ∨Elim, set up for it early. (A disjunction can be used with an easy rule if that disjunction forms the antecedent of a conditional or forms one side of a biconditional that you have on another line.

4. When aiming for ⊥, look for a negation you already have to use as one member of the contradictory pair of sentences.

5. If you have no idea what to do, you try applying any easy rules you can. Perhaps the results of this process will give you some ideas for other things to do.

6. When all else fails, assume the opposite of what you want, and aim for a contradiction.