

Extra Exercises for Chapter 3

Use Tarski's World for all exercises on this page.

H 3.1 (Building a world) Open the file **Sentences Cstern 0301**.

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|---------------------------------|---------------------------------------|
| 1. $\neg \text{Cube}(a)$ | 6. $\neg(d \neq b)$ |
| 2. $\neg \text{SameShape}(a,b)$ | 7. $\neg \text{SameCol}(a,e)$ |
| 3. $\neg \text{RightOf}(b,c)$ | 8. $\neg \neg \text{SameCol}(c,e)$ |
| 4. $\neg \neg \text{Tet}(b)$ | 9. $\neg \text{LeftOf}(a,c)$ |
| 5. $c \neq d$ | 10. $\neg \neg \text{SameShape}(c,d)$ |

Start a new world file and build a world where all these sentences are true. As you modify the world to make the later sentences true, make sure you have not accidentally falsified any of the earlier sentences. Save your world as **World Cstern 0301**. Submit both your world and your sentences to the GradeGrinder.

H 3.2 (Building a world) Open the file **Sentences Cstern 0302**.

- | | |
|--------------------------------------|-------------------------------------|
| 1. $\neg \text{Dodec}(a)$ | 6. $b \neq a$ |
| 2. $\neg \text{Tet}(a)$ | 7. $\neg(b \neq c)$ |
| 3. $\neg \text{Tet}(b)$ | 8. $\neg \neg \text{SameSize}(b,d)$ |
| 4. $\neg \text{FrontOf}(a,b)$ | 9. $\neg \text{SameShape}(d,c)$ |
| 5. $\neg \neg \text{SameShape}(b,e)$ | 10. $\neg \neg \text{SameRow}(c,d)$ |

Start a new world file and build a world where all these sentences are true. As you modify the world to make the later sentences true, make sure you have not accidentally falsified any of the earlier sentences. Save your world as **World Cstern 0302**. Submit both your world and your sentences to the GradeGrinder.

H 3.3 Open **Sentences Cstern 0303**. Show that the sentence

$\neg \text{Small}(a) \wedge \neg \text{Small}(b)$

is not a consequence of

$\neg(\text{Small}(a) \wedge \text{Small}(b))$

Do this by building a counterexample world in which the second sentence is true but the first is false. Save as **World Cstern 0303** and submit it to the GradeGrinder.

H 3.4 Open **Sentences Cstern 0304**. Show that the sentence

$\neg \text{Small}(a) \wedge \text{Cube}(a)$

is not a consequence of

$\neg(\text{Small}(a) \wedge \text{Cube}(a))$

Do this by building a counterexample world in which the second sentence is true but the first is false. Save as **World Cstern 0304** and submit it to the GradeGrinder.

H 3.5 Open **Sentences Cstern 0305**. Show that the sentence

$(\text{Small}(a) \wedge \text{Cube}(a)) \vee (\text{Small}(b) \wedge \text{Cube}(b))$

is not a consequence of

$(\text{Small}(a) \vee \text{Small}(b)) \wedge (\text{Cube}(a) \vee \text{Cube}(b))$

Do this by building a counterexample world in which the second sentence is true but the first is false. Save as **World Cstern 0305** and submit it to the GradeGrinder.

H 3.6 Open **Sentences Cstern 0306**. Show that the sentence

$\neg(\text{Large}(a) \vee \text{Tet}(a))$

is not a consequence of

$\neg \text{Large}(a) \vee \neg \text{Tet}(a)$

Do this by building a counterexample world in which the second sentence is true but the first is false. Save as **World Cstern 0306** and submit it to the GradeGrinder.

Extra Exercises for Chapter 4

Use Boole to construct truth tables and to indicate whether each sentence is TT-possible and whether each is a tautology. Build your own reference columns and fill them in yourself. (Do not have Boole do these steps for you.) Open file Table Cstern 0401 for H 4.1, etc.

$$\text{H 4.1} \quad (A \wedge B) \vee (\neg A \wedge \neg B)$$

$$\text{H 4.2} \quad (A \wedge B) \vee (\neg A \vee \neg B)$$

$$\text{H 4.3} \quad \neg(\neg(A \vee B) \wedge A)$$

$$\text{H 4.4} \quad \neg((\neg A \wedge \neg B) \wedge (A \vee B))$$

$$\text{H 4.5} \quad \neg(A \vee B) \wedge \neg(\neg A \wedge \neg B)$$

$$\text{H 4.6} \quad \neg(\neg A \vee \neg B) \wedge \neg(A \wedge B)$$

$$\text{H 4.7} \quad (A \vee B) \vee (\neg A \wedge \neg B)$$

$$\text{H 4.8} \quad \neg(A \vee B) \vee (A \wedge B)$$

Use Boole to construct truth tables and to indicate whether each pair of sentences is tautologically equivalent. Build your own reference columns and fill them in yourself. (Do not have Boole do these steps for you.) Open Table Cstern 0409 for exercise H 4.9, and Table Cstern 0410 for exercise H 4.10, etc.

$$\text{H 4.9} \quad \begin{array}{l} \neg(A \wedge B) \\ \neg A \wedge \neg B \end{array}$$

$$\text{H 4.10} \quad \begin{array}{l} \neg(A \wedge B) \\ \neg A \vee \neg B \end{array}$$

$$\text{H 4.11} \quad \begin{array}{l} \neg(A \vee B) \\ \neg A \vee \neg B \end{array}$$

Use the truth table method to determine whether the conclusion of each argument is a tautological consequence of the premises. Build your own reference columns and fill them in yourself. (Do not have Boole do these steps for you.) Open file Table Cstern 0412 for exercise H 4.12, etc.

$$\text{H 4.12} \quad \begin{array}{l} \text{Tet}(a) \vee \text{Small}(a) \\ \hline \text{Tet}(a) \\ \neg \text{Small}(a) \end{array}$$

$$\text{H 4.13} \quad \begin{array}{l} \text{Tet}(a) \wedge \text{Small}(a) \\ \hline (\text{Tet}(a) \vee \text{Cube}(a)) \wedge (\text{Small}(a) \vee \text{Large}(a)) \end{array}$$

$$\text{H 4.14} \quad \begin{array}{l} A \wedge \neg B \\ \hline B \vee C \\ A \wedge C \end{array}$$

$$\text{H 4.15} \quad \begin{array}{l} \neg(A \wedge B) \\ \hline B \vee C \\ \neg(\neg A \wedge C) \end{array}$$

$$\text{H 4.16} \quad \begin{array}{l} \neg(\neg A \vee B) \\ \hline B \vee C \\ A \wedge C \end{array}$$

$$\text{H 4.17} \quad \begin{array}{l} \neg A \vee B \\ \hline B \vee C \\ \neg(A \wedge C) \end{array}$$

$$\text{H 4.18} \quad \begin{array}{l} A \wedge (B \vee C) \\ \neg(A \wedge E) \\ \hline \neg(B \wedge C) \\ \neg E \wedge (\neg A \vee D) \end{array}$$

$$\text{H 4.19} \quad \begin{array}{l} A \vee (B \wedge C) \\ \hline \neg(C \vee (B \wedge D)) \\ \neg C \wedge A \end{array}$$

$$\text{H 4.20} \quad \begin{array}{l} (A \wedge B) \vee (C \wedge D) \\ \neg(A \wedge E) \\ \hline \neg(B \wedge C) \\ \neg E \wedge (\neg A \vee D) \end{array}$$

$$\text{H 4.21} \quad \begin{array}{l} (A \vee B) \wedge (C \vee D) \\ \neg(A \wedge (C \vee E)) \\ \hline \neg(\neg E \wedge \neg D) \\ B \vee (A \wedge (\neg E \wedge D)) \end{array}$$

Extra Problems for Chapter 7

For each pair of sentences, use Boole to determine whether (a) the two sentences are tautologically equivalent, (b) the second sentence is a tautological consequence of the first, and (c) the first sentence is a tautological consequence of the second. You may have Boole build the reference columns and fill them out for you. Open file **Table Cstern 0701** for the pair in H 7.1, etc., and **Table Cstern 0710** for the pair in H 7.10, etc.

$$\begin{array}{l} \text{H 7.1} \quad A \rightarrow B \\ \quad \quad B \rightarrow A \end{array}$$

$$\begin{array}{l} \text{H 7.3} \quad A \rightarrow B \\ \quad \quad \neg A \rightarrow \neg B \end{array}$$

$$\begin{array}{l} \text{H 7.5} \quad A \leftrightarrow B \\ \quad \quad \neg A \rightarrow \neg B \end{array}$$

$$\begin{array}{l} \text{H 7.7} \quad (A \wedge B) \rightarrow C \\ \quad \quad (A \rightarrow C) \wedge (B \rightarrow C) \end{array}$$

$$\begin{array}{l} \text{H 7.9} \quad (A \vee B) \rightarrow C \\ \quad \quad (A \rightarrow C) \wedge (B \rightarrow C) \end{array}$$

$$\begin{array}{l} \text{H 7.11} \quad A \rightarrow (B \wedge C) \\ \quad \quad (A \rightarrow B) \wedge (A \rightarrow C) \end{array}$$

$$\begin{array}{l} \text{H 7.2} \quad A \leftrightarrow B \\ \quad \quad B \leftrightarrow A \end{array}$$

$$\begin{array}{l} \text{H 7.4} \quad A \rightarrow \neg B \\ \quad \quad \neg(A \rightarrow B) \end{array}$$

$$\begin{array}{l} \text{H 7.6} \quad A \leftrightarrow B \\ \quad \quad (A \wedge B) \vee (\neg A \wedge \neg B) \end{array}$$

$$\begin{array}{l} \text{H 7.8} \quad (A \wedge B) \rightarrow C \\ \quad \quad (A \rightarrow C) \vee (B \rightarrow C) \end{array}$$

$$\begin{array}{l} \text{H 7.10} \quad (A \vee B) \rightarrow C \\ \quad \quad (A \rightarrow C) \vee (B \rightarrow C) \end{array}$$

$$\begin{array}{l} \text{H 7.12} \quad A \rightarrow (B \vee C) \\ \quad \quad (A \rightarrow B) \vee (A \rightarrow C) \end{array}$$

Use the truth table method (Boole) to determine whether each of the following arguments is tautologically valid -- that is, whether the conclusion is a tautological consequence of the premises. Open file **Table Cstern 0713** for H 7.13, etc.

$$\begin{array}{l|l} \text{H 7.13} & \begin{array}{l} A \rightarrow B \\ A \rightarrow \neg C \\ \hline B \vee C \\ \neg A \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.14} & \begin{array}{l} A \rightarrow B \\ (A \wedge B) \rightarrow C \\ \hline A \rightarrow C \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.15} & \begin{array}{l} (A \vee B) \leftrightarrow (A \wedge B) \\ \hline A \leftrightarrow B \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.16} & \begin{array}{l} (A \vee B) \rightarrow C \\ \neg A \leftrightarrow B \\ \hline C \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.17} & \begin{array}{l} A \leftrightarrow (B \vee C) \\ B \rightarrow (A \leftrightarrow \neg C) \\ \hline B \rightarrow \neg C \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.18} & \begin{array}{l} \neg(A \rightarrow B) \\ C \rightarrow (\neg B \rightarrow \neg A) \\ \hline \neg C \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.19} & \begin{array}{l} A \rightarrow (B \leftrightarrow (C \wedge D)) \\ C \rightarrow (\neg B \wedge A) \\ \hline \neg D \rightarrow \neg C \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.20} & \begin{array}{l} \neg A \rightarrow \neg(B \wedge C) \\ (C \vee D) \rightarrow B \\ \hline \neg(\neg A \wedge D) \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.21} & \begin{array}{l} (A \wedge B) \rightarrow (C \vee D) \\ \neg(A \wedge C) \\ \hline A \leftrightarrow (D \rightarrow B) \\ A \rightarrow (B \rightarrow D) \end{array} \end{array}$$

$$\begin{array}{l|l} \text{H 7.22} & \begin{array}{l} A \rightarrow \neg(B \vee C) \\ \neg D \rightarrow (A \leftrightarrow B) \\ \hline \neg(C \rightarrow D) \\ \neg A \wedge \neg D \end{array} \end{array}$$