Math 350, Exam Practice

1. Let $A \subset \mathbb{R}$. Show that the boundary of A is the same as $\overline{A} \cap \overline{A^C}$

2. Show that a closed subset of a compact set is compact.

3. Let C be compact, show that C is bounded.

4. Show that if A is closed, and x is a limit point of A, then $x \in A$.

5. Let F be the set constructed as follows. $F_0 = [0,1]$, $F_1 = [0,1/3] \cup [2/3,1]$, $F_2 = [0,1/9] \cup [2/9,1/3] \cup [2/3,7/9] \cup [8/9,1]$, etc. At each step F_n is a finite collection of closed intervals and F_{n+1} is obtained by removing the middle third from each of these intervals. Then $F = \bigcap_{n=0}^{\infty} F_n$. Prove that

   (a) F is closed.

   (b) F is compact.

   (c) The interior of F is empty.

   (d) F is uncountable.

6. Use the definition of continuity to show that $f(x) = \sqrt{x}$ is continuous on its domain.

7. Prove that
$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$
is continuous on its domain.

8. Prove that $f(x) = \sin x$ is uniformly continuous on $\mathbb{R}$.

9. Let f and g be uniformly continuous functions on $[a,b]$. Define $F(x) = \max\{f(x), g(x)\}$ and $G(x) = \min\{f(x), g(x)\}$. Prove that F and G are uniformly continuous.

10. Let $x_0 \in \mathbb{R}$ be fixed. For any set A define $d(x_0, A) = \inf_{x \in A} |x - x_0|$. This is the distance between x_0 and A. Show that if A is compact there exists an $x_1 \in A$ such that $d(x_0, A) = |x_0 - x_1|$. Give an example of a point x_0 and a set A, where this is not the case.

11. Let f be a strictly monotonic function on $[a, b]$. Prove that f is one-to-one, and that f^{-1} is also strictly monotonic.