MATH 450 — Assessment of Retention of Math 350 — Answers in bold

This is not a test. This is only an assessment to find out how much you remember from Math 350 it counts for nothing.

1. Please let me know when and where you took Math 350 or its equivalent?

2. Which of the following is a correct **definition** of compactness? A set C is compact if and only if

   (a) it has an open subcover.

   (b) it is closed and bounded. (true in $\mathbb{R}^n$ but not a definition)

   (c) **every open cover has a finite subcover.**

   (d) there exist an open cover with a finite subcovers.

   (e) every open cover is finite.

3. Of the following statements which are correct. A function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if

   (a) for every open set O, $f(O)$ is also open.

   (b) **for every $y \in \mathbb{R}$ and every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ for all $x \in (y - \delta, y + \delta)$.**

   (c) **for every open set O, $f^{-1}(O)$ is open.**

   (d) it is differentiable.

   (e) for every $\epsilon > 0$ there is a $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ for all x, y with $|x - y| < \delta$.(thats uniformly continuous)

4. Of the following statements which are correct. A sequence $\{a_n\}$ is convergent if and only if

   (a) **there exists a number L such that for every $\epsilon > 0$ there is a number N such that $|a_n - L| < \epsilon$ for all $n \geq N$.**

   (b) **it is a Cauchy sequence.** (holds for all complete metric spaces)

   (c) there exists a number L such that for every $\epsilon > 0$ the interval $(L - \epsilon, L + \epsilon)$ contains infinitely many terms of the sequence.

   (d) **there exists a number L such that for every $\epsilon > 0$ the interval $(L - \epsilon, L + \epsilon)$ contains all but finitely many terms of the sequence.**

   (e) it is bounded and monotonic.

5. Of the following statements which are correct. The function $f : [a, b] \to \mathbb{R}$ is Riemann integrable, if

   (a) **it is continuous.**

   (b) **for every $\epsilon > 0$ there is a partition P such that $\overline{S}(P; f) - \underline{S}(P; f) < \epsilon$.**

   (c) **it is monotonic.**

   (d) for all $N \in \mathbb{N}$, and $x_n = a + \frac{b-a}{N}$ the sum $\sum_{n=1}^{N} f(x_n)\frac{b-a}{N}$ converges to a finite limit.

   (e) **if the set of discontinuities of f has measure zero.**

6. Of the following statements which are correct. Let S be the set of differentiable functions on (a, b). The S is

   (a) **a real vectorspace.**

   (b) finite dimensional.

   (c) **a commutative ring.**

   (d) a field.

   (e) none of the above.