

Math 350 - Last Assignment

1. Let f be continuous on $[a, b]$ and g be non-negative and Riemann integrable. Moreover, let m, M be such that $m \leq f(x) \leq M$ for all $x \in [a, b]$. Prove that there is a $\xi \in [a, b]$ such that

$$\int_a^b fg = m \int_a^\xi g + M \int_\xi^b g.$$

2. Let f be Riemann integrable. Use the upper and lower sums and the integrability criterium to show that f^2 is Riemann integrable.
3. Let f and g be Riemann integrable. Use the previous problem to Show that $(f + g)^2$ is Riemann integrable and then prove that fg is Riemann integrable.