16.1 OPTION VALUATION: INTRODUCTION

Option Values
- Intrinsic value - profit that could be made if the option was immediately exercised
  - Call: stock price - exercise price
  - Put: exercise price - stock price
- Time value - the difference between the option price and the intrinsic value

Figure 16.1 Call Option Value Before Expiration

Determinants of Call Option Values
- Stock price
- Exercise price
- Volatility of the stock price
- Time to expiration
- Interest rate
- Dividend rate of the stock

Table 16.1 Determinants of Call Option Values
16.2 BINOMIAL OPTION PRICING

Binomial Option Pricing: Text Example

Stock Price

- 100
- 90
- 120

Call Option Value

- 10
- 0

Binomial Option Pricing: Text Example (cont.)

Alternative Portfolio
Buy 1 share of stock at $100
Borrow $81.82 (10% Rate)
Net outlay $18.18

Payoff
Value of Stock 90 120
Repay loan -90 -90
Net Payoff 0 30

Payoff Structure is exactly 3 times the Call

3C = $18.18
C = $ 6.06

Another View of Replication of Payoffs and Option Values

Alternative Portfolio - one share of stock and 3 calls written (X = 110)
Portfolio is perfectly hedged
Stock Value 90 120
Call Obligation 0 -30
Net payoff 90 90

Hence 100 - 3C = 81.82 or C = 6.06

Generalizing the Two-State Approach
16.3 BLACK-SCHOLES OPTION VALUATION

\[ C_o = S_o e^{-rT} N(d_1) - X e^{rT} N(d_2) \]
\[ d_1 = \left[ \ln\left(\frac{S_o}{X}\right) + (r - \delta + \sigma^2/2)T \right] / (\sigma \sqrt{T}) \]
\[ d_2 = d_1 - (\sigma \sqrt{T}) \]

where
\[ C_o = \text{Current call option value.} \]
\[ S_o = \text{Current stock price} \]
\[ N(d) = \text{probability that a random draw from a normal dist. will be less than d.} \]

X = Exercise price
\( \delta \) = Annual dividend yield of underlying stock
\( e \approx 2.71828 \), the base of the natural log
\( r \) = Risk-free interest rate (annualizes continuously compounded with the same maturity as the option)
\( T \) = Time to maturity of the option in years.
\( \ln \) = Natural log function
\( \sigma \) = Standard deviation of annualized cont. compounded rate of return on the stock
Call Option Example

\[ S_0 = 100 \quad X = 95 \]
\[ r = .10 \quad T = .25 \text{ (quarter)} \]
\[ \sigma = .50 \quad \delta = 0 \]
\[ d_1 = \frac{\ln(100/95) + (.10 - .0 + (.50^2/2))}{.50^{1/2}} \approx .43 \]
\[ d_2 = .43 - (.50^{1/2}) \approx .18 \]

Probabilities from Normal Distribution

\[ N (.43) = .6664 \]
Table 17.2
\[ N(d) \]
\[ .42 \quad .6628 \]
\[ .43 \quad .6664 \quad \text{Interpolation} \]
\[ .44 \quad .6700 \]

Probabilities from Normal Distribution

\[ N (.18) = .5714 \]
Table 17.2
\[ d \quad N(d) \]
\[ .16 \quad .5636 \]
\[ .18 \quad .5714 \]
\[ .20 \quad .5793 \]

Call Option Value

\[ C_0 = S_0 e^{\sigma T N(d_1)} - X e^{r T N(d_2)} \]
\[ C_0 = 100 \times .6664 - 95 \times e^{.10 \times .25} \times .5714 \]
\[ C_0 = 13.70 \]

Implied Volatility
Using Black-Scholes and the actual price of the option, solve for volatility.
Is the implied volatility consistent with the stock?

Figure 16.5 Implied Volatility of the S&P 500 (VIX Index)

Put-Call Parity Relationship

<table>
<thead>
<tr>
<th>( S_T \leq X )</th>
<th>( S_T &gt; X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff for Call Owned</td>
<td>0</td>
</tr>
<tr>
<td>Payoff for Put Written</td>
<td>((X - S_T))</td>
</tr>
<tr>
<td>Total Payoff</td>
<td>( S_T - X )</td>
</tr>
</tbody>
</table>
Arbitrage & Put Call Parity

Since the payoff on a combination of a long call and a short put are equivalent to leveraged equity, the prices must be equal.

\[ C - P = S_0 - X / (1 + r)^T \]

If the prices are not equal arbitrage will be possible.

Put Call Parity - Disequilibrium Example

Stock Price = 110  Call Price = 17
Put Price = 5       Risk Free = 5%
Maturity = .5 yr    Exercise (X) = 105
\[ C - P > S_0 - X / (1 + r)^T \]
14 - 5 > 110 - (105 e^{.05 \times .5})
9 > 7.59
Since the leveraged equity is less expensive, acquire the low cost alternative and sell the high cost alternative.

Example 16.3 Put-Call Parity

Using the sample data
\[ P = 13.70 + 95 e^{-10 \times .25} \]
\[ P = 6.35 \]

Put Option Valuation

\[ P = X e^{rt} [1-N(d_2)] - S_0 e^{-rt} [1-N(d_1)] \]
Using the sample data
\[ P = 95 e^{-.10 \times .25} (1-.5714) - 100 (1-.6664) \]
\[ P = 6.35 \]
16.4 USING THE BLACK-SCHOLES FORMULA

Black-Scholes Formula

- Hedging: Hedge ratio or delta
  The number of stocks required to hedge against the price risk of holding one option
  \[
  \text{Call} = N(d_1) \\
  \text{Put} = N(d_1) - 1
  \]

- Option Elasticity
  Percentage change in the option's value given a 1% change in the value of the underlying security

Portfolio Insurance - Protecting Against Declines in Stock Value

- Buying Puts - results in downside protection with unlimited upside potential
- Limitations
  - Tracking errors if indexes are used for the puts
  - Maturity of puts may be too short
  - Hedge ratios or deltas change as stock values change

Figure 16.7 Call Option Value and Hedge Ratio

Figure 16.8 Profit on a Protective Put Strategy

Figure 16.9 Hedge-Ratios Change as the Stock Price Fluctuates
16.5 EMPIRICAL EVIDENCE

Empirical Tests of Black-Scholes Option Pricing

- Implied volatility varies with exercise price
- If the model were accurate, implied volatility should be the same for all options with the same expiration date
- Implied volatility steadily falls as the exercise price rises

Figure 16.10 Implied Volatility as a Function of Exercise Price