CHAPTER 16

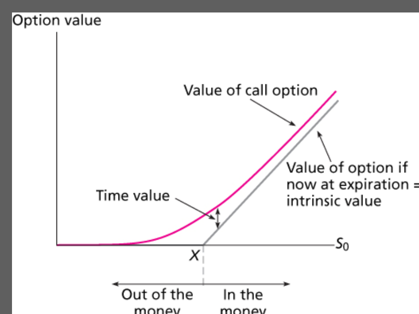
Option Valuation

16.1 OPTION VALUATION: INTRODUCTION

Option Values

- Intrinsic value - profit that could be made if the option was immediately exercised
  - Call: stock price - exercise price
  - Put: exercise price - stock price
- Time value - the difference between the option price and the intrinsic value

Figure 16.1 Call Option Value Before Expiration



Determinants of Call Option Values

- Stock price
- Exercise price
- Volatility of the stock price
- Time to expiration
- Interest rate
- Dividend rate of the stock

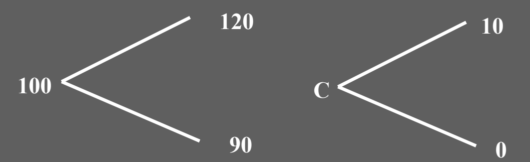
Table 16.1 Determinants of Call Option Values

TABLE 16.1 Determinants of call option values

If This Variable Increases	The Value of a Call Option
Stock price, S	Increases
Exercise price, X	Decreases
Volatility, σ	Increases
Time to expiration, T	Increases
Interest rate, r_f	Increases
Dividend payouts	Decreases

16.2 BINOMIAL OPTION PRICING

Binomial Option Pricing: Text Example



Stock Price

Call Option Value if X = 110

Binomial Option Pricing: Text Example (cont.)

Alternative Portfolio

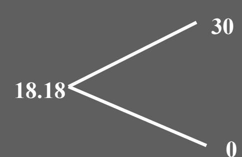
Buy 1 share of stock at $100

Borrow $81.82 (10% Rate)

Net outlay $18.18

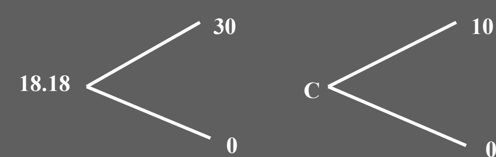
Payoff

Value of Stock	90	120
Repay loan	- 90	-90
Net Payoff	0	30



Payoff Structure is exactly 3 times the Call

Binomial Option Pricing: Text Example (cont.)



3C = $18.18

C = $ 6.06

Another View of Replication of Payoffs and Option Values

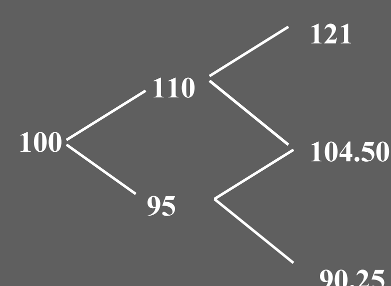
Alternative Portfolio - one share of stock and 3 calls written (X = 110)

Portfolio is perfectly hedged

Stock Value	90	120
Call Obligation	0	- 30
Net payoff	90	90

Hence 100 - 3C = 81.82 or C = 6.06

Generalizing the Two-State Approach



Generalizing the Two-State Approach

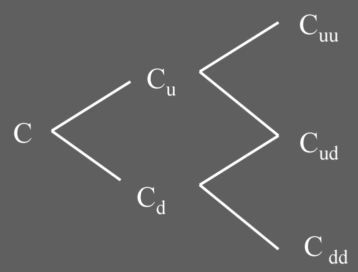
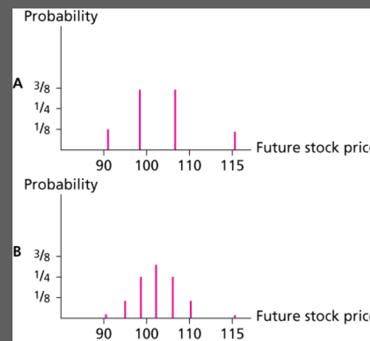


Figure 16.2 Probability Distributions



16.3 BLACK-SCHOLES OPTION VALUATION

Black-Scholes Option Valuation

$C_o = S_o e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$

$d_1 = [\ln(S_o/X) + (r - \delta + \sigma^2/2)T] / (\sigma T^{1/2})$

$d_2 = d_1 - (\sigma T^{1/2})$

where

C_o = Current call option value.

S_o = Current stock price

N(d) = probability that a random draw from a normal dist. will be less than d.

Black-Scholes Option Valuation

X = Exercise price.

δ = Annual dividend yield of underlying stock

e = 2.71828, the base of the natural log

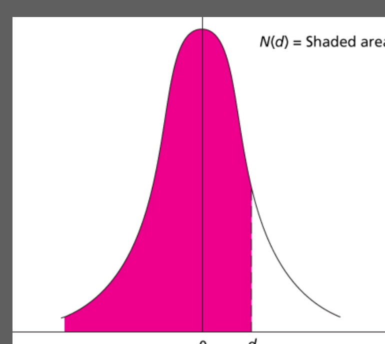
r = Risk-free interest rate (annualizes continuously compounded with the same maturity as the option.

T = time to maturity of the option in years.

ln = Natural log function

σ = Standard deviation of annualized cont. compounded rate of return on the stock

Figure 16.3 A Standard Normal Curve



Call Option Example

$S_o = 100$ $\quad X = 95$

$r = .10$ $\quad T = .25$ (quarter)

$\sigma = .50$ $\quad \delta = 0$

$d_1 = [\ln(100/95)+(.10-0+(.5^{2}/2))]/(.5\ .25^{1/2})$

$= .43$

$d_2 = .43 - ((.5)(.25)^{1/2}$

$= .18$

Probabilities from Normal Distribution

N (.43) = .6664

Table 17.2

d	N(d)	
.42	.6628	
.43	.6664	Interpolation
.44	.6700	

Probabilities from Normal Distribution

N (.18) = .5714

Table 17.2

d	N(d)
.16	.5636
.18	.5714
.20	.5793

Call Option Value

$C_o = S_o e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$

$C_o = 100 \text{ X } .6664 - 95\ e^{-.10 \text{ X } .25} \text{ X } .5714$

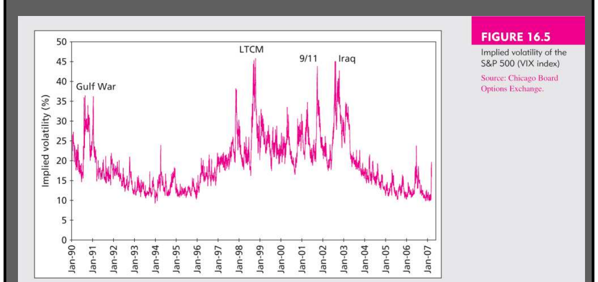
$C_o = 13.70$

<u>Implied Volatility</u>

Using Black-Scholes and the actual price of the option, solve for volatility.

Is the implied volatility consistent with the stock?

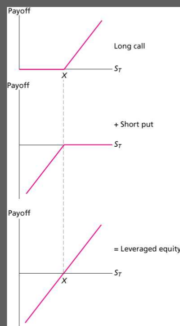
Figure 16.5 Implied Volatility of the S&P 500 (VIX Index)



Put-Call Parity Relationship

	$S_T \leq X$	$S_T > X$
Payoff for Call Owned	**0**	$S_T - X$
Payoff for Put Written	$-(X - S_T)$	**0**
Total Payoff	$S_T - X$	$S_T - X$

Figure 16.6 The Payoff Pattern of a Long Call – Short Put Position



Arbitrage & Put Call Parity

Since the payoff on a combination of a long call and a short put are equivalent to leveraged equity, the prices must be equal.

$$C - P = S_0 - X / (1 + r_f)^T$$

If the prices are not equal arbitrage will be possible

Put Call Parity - Disequilibrium Example

Stock Price = 110   Call Price = 17
Put Price = 5   Risk Free = 5%
Maturity = .5 yr   Exercise (X) = 105

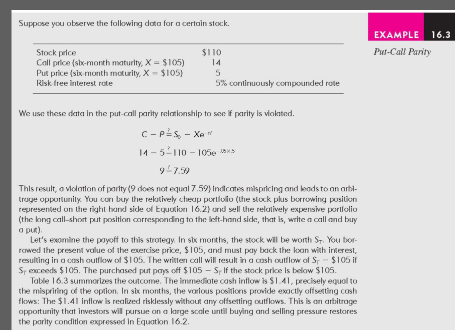
$$C - P > S_0 - X / (1 + r_f)^T$$

$$14 - 5 > 110 - (105\, e^{(-.05 \times .5)})$$

$$9 > 7.59$$

Since the leveraged equity is less expensive; acquire the low cost alternative and sell the high cost alternative

Example 16.3 Put-Call Parity



EXAMPLE 16.3

Put-Call Parity

Suppose you observe the following data for a certain stock.

Stock price	\$110
Call price (six-month maturity, X = \$105)	14
Put price (six-month maturity, X = \$105)	5
Risk-free interest rate	5% continuously compounded rate

We use these data in the put-call parity relationship to see if parity is violated.

$$C - P \stackrel{?}{=} S_0 - Xe^{-rT}$$

$$14 - 5 \stackrel{?}{=} 110 - 105e^{-.05 \times .5}$$

$$9 \stackrel{?}{=} 7.59$$

This result, a violation of parity (9 does not equal 7.59) indicates mispricing and leads to an arbitrage opportunity. You can buy the relatively cheap portfolio (the stock plus borrowing position represented on the right-hand side of Equation 16.2) and sell the relatively expensive portfolio (the long call–short put position corresponding to the left-hand side, that is, write a call and buy a put).

Let's examine the payoff to this strategy. In six months, the stock will be worth S_T. You borrowed the present value of the exercise price, \$105, and must pay back the loan with interest, resulting in a cash outflow of \$105. The written call will result in a cash outflow of S_T − \$105 if S_T exceeds \$105. The purchased put pays off \$105 − S_T if the stock price is below \$105.

Table 16.3 summarizes the outcome. The immediate cash inflow is \$1.41, precisely equal to the mispriring of the option. In six months, the various positions provide exactly offsetting cash flows: The \$1.41 inflow is realized risklessly without any offsetting outflows. This is an arbitrage opportunity that investors will pursue on a large scale until buying and selling pressure restores the parity condition expressed in Equation 16.2.

Put Option Valuation

$$P = Xe^{-rT}\,[1-N(d_2)] - S_0 e^{-\delta T}\,[1-N(d_1)]$$

Using the sample data

$$P = \$95e^{(-.10X.25)}(1-.5714) - \$100\,(1-.6664)$$

$$P = \$6.35$$

Put Option Valuation: Using Put-Call Parity

$$P = C + PV(X) - S_o + PV(Div)$$
$$= C + Xe^{-rT} - S_o + 0$$

Using the example data

C = 13.70   X = 95   S = 100
r = .10   T = .25

$$P = 13.70 + 95\, e^{-.10 \times .25} - 100 + 0$$

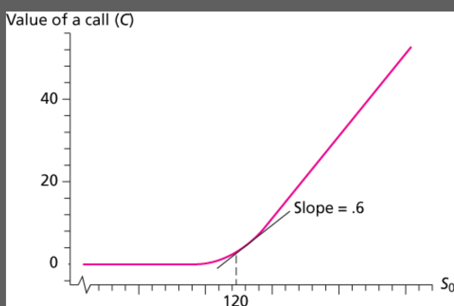
$$P = 6.35$$

16.4 USING THE BLACK-SCHOLES FORMULA

Black-Scholes Formula

- Hedging: Hedge ratio or delta
  The number of stocks required to hedge against the price risk of holding one option
  Call = N (d_1)
  Put = N (d_1) - 1
- Option Elasticity
  Percentage change in the option's value given a 1% change in the value of the underlying security

Figure 16.7 Call Option Value and Hedge Ratio



Portfolio Insurance - Protecting Against Declines in Stock Value

- Buying Puts - results in downside protection with unlimited upside potential
- Limitations
  - Tracking errors if indexes are used for the puts
  - Maturity of puts may be too short
  - Hedge ratios or deltas change as stock values change

Figure 16.8 Profit on a Protective Put Strategy

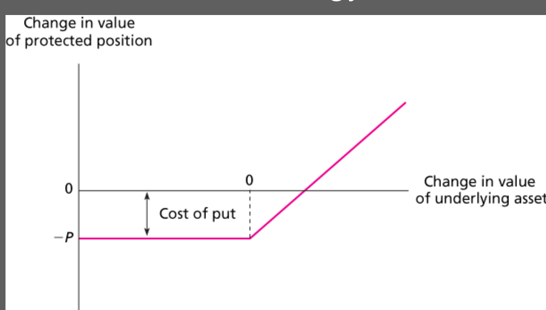
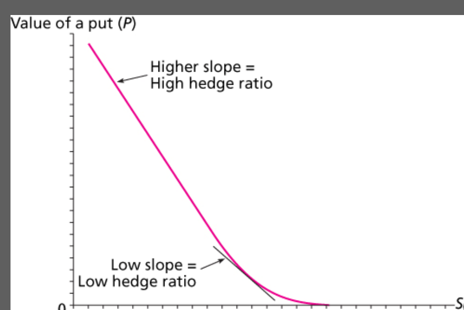


Figure 16.9 Hedge-Ratios Change as the Stock Price Fluctuates



16.5 EMPIRICAL EVIDENCE

Empirical Tests of Black-Scholes Option Pricing

- Implied volatility varies with exercise price
  - If the model were accurate, implied volatility should be the same for all options with the same expiration date
  - Implied volatility steadily falls as the exercise price rises

Figure 16.10 Implied Volatility as a Function of Exercise Price