11.1 INTEREST RATE RISK

Interest Rate Sensitivity

- Inverse relationship between price and yield
- An increase in a bond’s yield to maturity results in a smaller price decline than the gain associated with a decrease in yield
- Long-term bonds tend to be more price sensitive than short-term bonds

Interest Rate Sensitivity (cont)

- Sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases
- Interest rate risk is inversely related to bond’s coupon rate
- Sensitivity of a bond’s price to a change in its yield is inversely related to the yield to maturity at which the bond currently is selling

Figure 11.1 Change in Bond Price as a Function of YTM

Duration

- A measure of the effective maturity of a bond
- The weighted average of the times until each payment is received, with the weights proportional to the present value of the payment
- Duration is shorter than maturity for all bonds except zero coupon bonds
- Duration is equal to maturity for zero coupon bonds
Figure 11.2 Cash Flows of 8-yr Bond with 9% annual coupon and 10% YTM

Duration Calculation

\[ \frac{W_t}{P} = \frac{[CF_t/(1+y)^t]}{Price} \]

\[ D = \sum_{t=1}^{T} t \times W_t \]

\[ CF_t = \text{Cash Flow for period } t \]

Duration Calculation

<table>
<thead>
<tr>
<th>8% Bond</th>
<th>Time (years)</th>
<th>Payment</th>
<th>PV of CF (10%)</th>
<th>Weight</th>
<th>C1 X C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>80</td>
<td>72.727</td>
<td>.0765</td>
<td>.0765</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>80</td>
<td>66.116</td>
<td>.0690</td>
<td>.1392</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1080</td>
<td>811.420</td>
<td>.8530</td>
<td>2.5617</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>1080</td>
<td>850.263</td>
<td>1.0000</td>
<td>2.7774</td>
</tr>
</tbody>
</table>

Duration/Price Relationship

- Price change is proportional to duration and not to maturity
- \( \Delta P/P = -D \times [\Delta y / (1+y)] \)
- \( D' = \text{modified duration} \)
- \( D' = D / (1+y) \)
- \( \Delta P/P = -D' \times \Delta y \)

11.2 PASSIVE BOND MANAGEMENT
Immunization

- Passive management
  - Net worth immunization
  - Target date immunization

Figure 11.4 Growth of Invested Funds

Figure 11.5 Immunization

Cash Flow Matching and Dedication

- Automatically immunizes a portfolio from interest rate movements
  - Cash flow from the bond and the obligation exactly offset each other
- Not widely pursued
- Sometimes not even possible

11.3 CONVEXITY

Limitations of Duration

- Duration is only an approximation
- Duration asserts that the percentage price change is directly proportional to the change in the bond’s yield
- Underestimates the increase in bond prices when yield falls
- Overestimates the decline in price when the yield rises
Pricing Error from Convexity

Correction for Convexity

Modify the pricing equation:

\[
\frac{\Delta P}{P} = -D \times \Delta y + \sum_{t=1}^{\infty} \text{Convexity} \times (\Delta y)^2
\]

Convexity is Equal to:

\[
\frac{1}{P} \times (1 + y)^2 \sum_{t=1}^{\infty} \left[ \frac{CF_t}{(1 + y)^t} \right] (t^2 + t)
\]

Where: \( CF_t \) is the cash flow (interest and/or principal) at time \( t \).

Figure 11.6 Bond Price Convexity

Figure 11.7 Convexity of Two Bonds

11.4 ACTIVE BOND MANAGEMENT

Swapping Strategies
- Substitution swap
- Intermarket swap
- Rate anticipation swap
- Pure yield pickup
- Tax swap
Horizon Analysis

Analyst selects a particular investment period and predicts bond yields at the end of that period.

Contingent Immunization

- Allow the managers to actively manage until the bond portfolio falls to a threshold level.
- Once the threshold value is hit, the manager must then immunize the portfolio.
- Active with a floor loss level.

Figure 11.8 Contingent Immunization