CHAPTER 6

Efficient Diversification

6.1 DIVERSIFICATION AND PORTFOLIO RISK

Diversification and Portfolio Risk

- Market risk
  - Systematic or Nondiversifiable
- Firm-specific risk
  - Diversifiable or nonsystematic

Figure 6.1 Portfolio Risk as a Function of the Number of Stocks

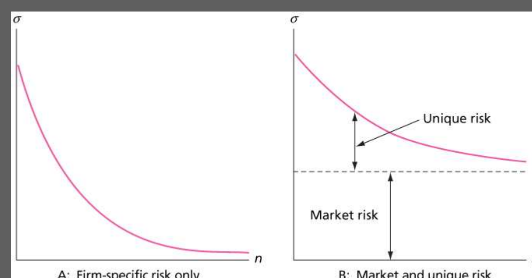
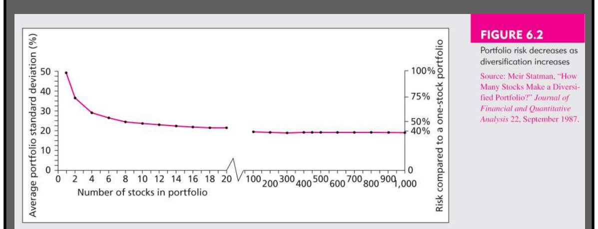


Figure 6.2 Portfolio Risk as a Function of Number of Securities



6.2 ASSET ALLOCATION WITH TWO RISKY ASSETS

Covariance and Correlation

- Portfolio risk depends on the correlation between the returns of the assets in the portfolio
- Covariance and the correlation coefficient provide a measure of the returns on two assets to vary

Two Asset Portfolio Return – Stock and Bond

$$r_p = w_B r_B + w_S r_S$$

r_P = Portfolio Return

w_B = Bond Weight

r_B = Bond Return

w_S = Stock Weight

r_S = Stock Return

Covariance and Correlation Coefficient

- Covariance:

$$Cov(r_S, r_B) = \sum_{i=1}^{S} p(i)\left[r_S(i) - \overline{r_S}\right]\left[r_B(i) - \overline{r_B}\right]$$

- Correlation Coefficient:

$$\rho_{SB} = \frac{Cov(r_S, r_B)}{\sigma_S \sigma_B}$$

Correlation Coefficients: Possible Values

Range of values for $\rho_{1,2}$

$$-1.0 \le \rho \le 1.0$$

If ρ = 1.0, the securities would be perfectly positively correlated

If ρ = - 1.0, the securities would be perfectly negatively correlated

Two Asset Portfolio St Dev – Stock and Bond

$$\sigma_p^2 = w_B^2 \sigma_B^2 + w_S^2 \sigma_S^2 + 2 w_B w_S \sigma_S \sigma_B \rho_{B,S}$$

σ_p^2 = Portfolio Variance

$\sqrt{\sigma_p^2}$ = Portfolio Standard Deviation

In General, For an n-Security Portfolio:

r_p = Weighted average of the n securities

σ_p^2 = (Consider all pair-wise covariance measures)

Three Rules of Two-Risky-Asset Portfolios

- Rate of return on the portfolio:

$$r_P = w_B r_B + w_S r_S$$

- Expected rate of return on the portfolio:

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

Three Rules of Two-Risky-Asset Portfolios

- Variance of the rate of return on the portfolio:

$$\sigma_P^2 = (w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S)\rho_{BS}$$

Numerical Text Example: Bond and Stock Returns (Page 169)

Returns
  Bond = 6% Stock = 10%
Standard Deviation
  Bond = 12% Stock = 25%
Weights
  Bond = .5 Stock = .5
Correlation Coefficient
  (Bonds and Stock) = 0

Numerical Text Example: Bond and Stock Returns (Page 169)

Return = 8%
  .5(6) + .5 (10)

Standard Deviation = 13.87%
  $[(.5)^2 (12)^2 + (.5)^2 (25)^2 + \ldots$
  $2 (.5) (.5) (12) (25) (0)]^{½}$
  $[192.25]^{½} = 13.87$

Figure 6.3 Investment Opportunity Set for Stocks and Bonds

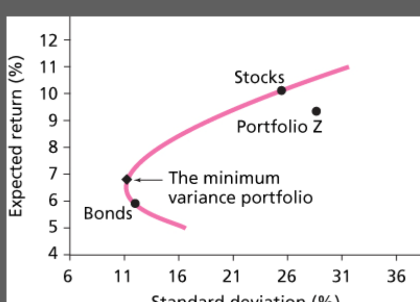
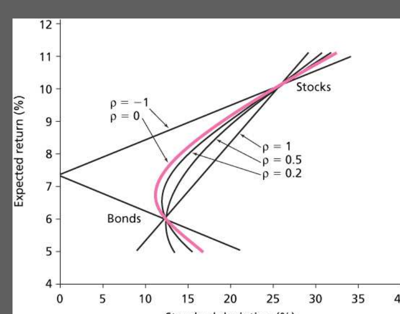


Figure 6.4 Investment Opportunity Set for Stocks and Bonds with Various Correlations

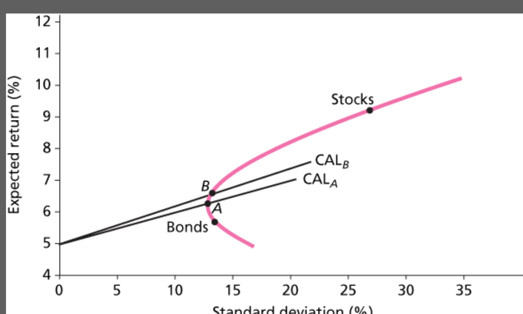


6.3 THE OPTIMAL RISKY PORTFOLIO WITH A RISK-FREE ASSET

Extending to Include Riskless Asset

- The optimal combination becomes linear
- A single combination of risky and riskless assets will dominate

Figure 6.5 Opportunity Set Using Stocks and Bonds and Two Capital Allocation Lines



Dominant CAL with a Risk-Free Investment (F)

CAL(O) dominates other lines -- it has the best risk/return or the largest slope

Slope = $\frac{E(r_A) - r_f}{\sigma_A}$

Dominant CAL with a Risk-Free Investment (F)

$$\frac{E(r_P) - r_f}{\sigma_P} > \frac{E(r_A) - r_f}{\sigma_A}$$

Regardless of risk preferences, combinations of O & F dominate

Figure 6.6 Optimal Capital Allocation Line for Bonds, Stocks and T-Bills

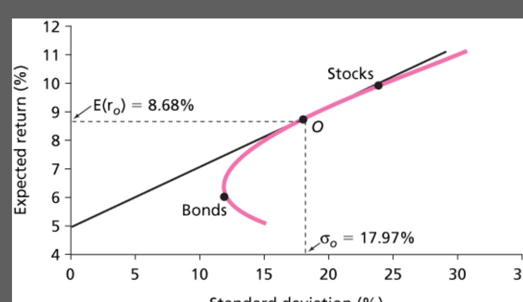


Figure 6.7 The Complete Portfolio

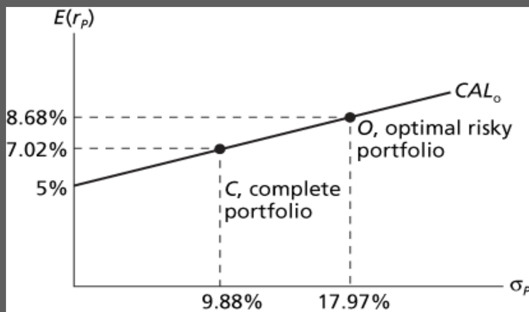
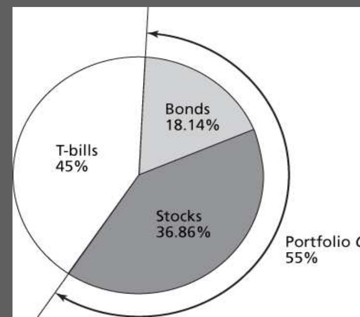


Figure 6.8 The Complete Portfolio – Solution to the Asset Allocation Problem



6.4 EFFICIENT DIVERSIFICATION WITH MANY RISKY ASSETS

Extending Concepts to All Securities

- The optimal combinations result in lowest level of risk for a given return
- The optimal trade-off is described as the efficient frontier
- These portfolios are dominant

Figure 6.9 Portfolios Constructed from Three Stocks A, B and C

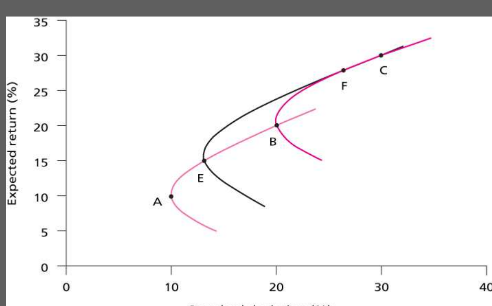
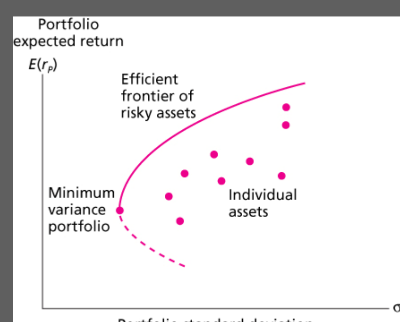


Figure 6.10 The Efficient Frontier of Risky Assets and Individual Assets



6.5 A SINGLE-FACTOR ASSET MARKET

Single Factor Model

$$R_i = E(R_i) + \beta_i M + e_i$$

β_i = index of a securities' particular return to the factor

M = unanticipated movement commonly related to security returns

E_i = unexpected event relevant only to this security

Assumption: a broad market index like the S&P500 is the common factor

Specification of a Single-Index Model of Security Returns

- Use the S&P 500 as a market proxy
- Excess return can now be stated as:

$$R_i = \alpha + \beta_i R_M + e$$

  - This specifies the both market and firm risk

Figure 6.11 Scatter Diagram for Dell

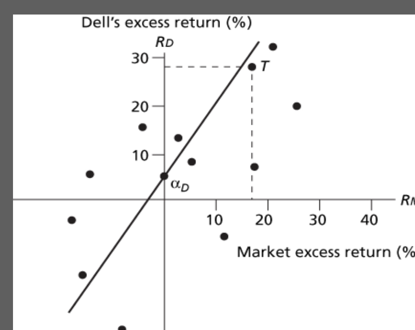
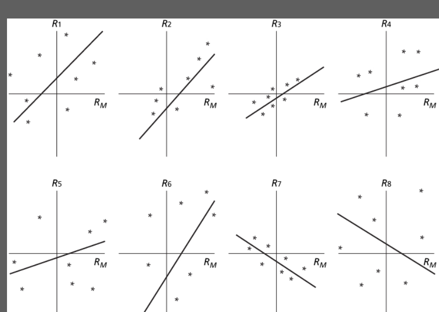


Figure 6.12 Various Scatter Diagrams



Components of Risk

- Market or systematic risk: risk related to the macro economic factor or market index
- Unsystematic or firm specific risk: risk not related to the macro factor or market index
- Total risk = Systematic + Unsystematic

Measuring Components of Risk

$\sigma_i^2 = \beta_i^2\ \sigma_m^2 + \sigma^2(e_i)$

where;

σ_i^2 = total variance

$\beta_i^2\ \sigma_m^2$ = systematic variance

$\sigma^2(e_i)$ = unsystematic variance

Examining Percentage of Variance

Total Risk = Systematic Risk + Unsystematic Risk

Systematic Risk/Total Risk = ρ^2

$\beta_i^2\ \sigma_m^2 / \sigma^2 = \rho^2$

$\beta_i^2\ \sigma_m^2 / \beta_i^2\ \sigma_m^2 + \sigma^2(e_i) = \rho^2$

Advantages of the Single Index Model

- Reduces the number of inputs for diversification
- Easier for security analysts to specialize

6.6 RISK OF LONG-TERM INVESTMENTS

Are Stock Returns Less Risky in the Long Run?

- Consider a 2-year investment

$$\text{Var (2-year total return)} = Var(r_1 + r_2$$
$$= Var(r_1) + Var(r_2) + 2Cov(r_1, r_2)$$
$$= \sigma^2 + \sigma^2 + 0$$
$$= 2\sigma^2 \text{ and standard deviation of the return is } \sigma\sqrt{2}$$

- Variance of the 2-year return is double of that of the one-year return and σ is higher by a multiple of the square root of 2

Are Stock Returns Less Risky in the Long Run?

- Generalizing to an investment horizon of *n* years and then annualizing:

$$\text{Var(n-year total return)} = n\sigma^2$$
$$\text{Standard deviation (}n\text{-year total return)} = \sigma\sqrt{n}$$
$$\sigma\text{(annualized for an } n \text{ - year investment)} = \frac{1}{n}\sigma\sqrt{n} = \frac{\sigma}{\sqrt{n}}$$

The Fly in the 'Time Diversification' Ointment

- Annualized standard deviation is only appropriate for short-term portfolios
- Variance grows linearly with the number of years
- Standard deviation grows in proportion to $\sqrt{n}$

The Fly in the 'Time Diversification' Ointment

- To compare investments in two different time periods:
  - Risk of the total (end of horizon) rate of return
  - Accounts for magnitudes and probabilities