CHAPTER 5
Risk and Return: Past and Prologue

5.1 RATES OF RETURN

Holding Period Return

\[ HPR = \frac{P_1 - P_0 + D_1}{P_0} \]

- \( P_0 \) = Beginning Price
- \( P_1 \) = Ending Price
- \( D_1 \) = Cash Dividend

Rates of Return: Single Period Example

Ending Price = 24
Beginning Price = 20
Dividend = 1

\[ HPR = \frac{(24 - 20 + 1)}{20} = 25\% \]

Measuring Investment Returns Over Multiple Periods

- May need to measure how a fund performed over a preceding five-year period
- Return measurement is more ambiguous in this case

Rates of Return: Multiple Period Example Text (Page 128)

Data from Table 5.1

<table>
<thead>
<tr>
<th>Year</th>
<th>Assets(Beg.)</th>
<th>HPR</th>
<th>TA (Before)</th>
<th>Net Flows</th>
<th>End Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>.10</td>
<td>.25</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>.25</td>
<td>(.20)</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>.25</td>
<td>(.20)</td>
<td>1.6</td>
<td>.8</td>
</tr>
<tr>
<td>4</td>
<td>.8</td>
<td>.25</td>
<td>.25</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>


Geometric Averaging

\[ r_g = \left( \frac{1}{n} \sum_{i=1}^{n} (1 + r_i) \right)^{1/n} - 1 \]

1. \[ (1.1) (1.25) (.8) (1.25) \]
2. \[ 1.5150 \]
3. \[ 0.10 \text{ or } 10\% \]
4. \[ 0.0829 = 8.29\% \]

Dollar Weighted Returns

**Internal Rate of Return (IRR)** - the discount rate that results in present value of the future cash flows being equal to the investment amount

- Considers changes in investment
- Initial Investment is an outflow
- Ending value is considered as an inflow
- Additional investment is a negative flow
- Reduced investment is a positive flow

\[ \text{IRR} = \left( \frac{\text{Ending value} - \text{Initial investment}}{\text{Initial investment}} \right)^{1/\text{Periods per yr}} - 1 \]

Example: monthly return of 1%

\[ \text{EAR} = \left( (1 + \text{rate for period})^{\text{Periods per yr}} - 1 \right) \]

\[ \text{APR} = 1\% \times 12 = 12\% \]

\[ \text{EAR} = (1.01)^{12} - 1 = 12.68\% \]

Quoting Conventions

**APR** = annual percentage rate

\[ \text{APR} = \text{(periods in year)} \times \text{(rate for period)} \]

**EAR** = effective annual rate

\[ \text{EAR} = \left( (1 + \text{rate for period})^{\text{Periods per yr}} - 1 \right) \]

Example: monthly return of 1%

\[ \text{APR} = 1\% \times 12 = 12\% \]

\[ \text{EAR} = (1.01)^{12} - 1 = 12.68\% \]

5.2 RISK AND RISK PREMIUMS

1) Mean: most likely value
2) Variance or standard deviation
3) Skewness

* If a distribution is approximately normal, the distribution is described by characteristics 1 and 2
Symmetric distribution

Skewed Distribution: Large Negative
Returns Possible

Skewed Distribution: Large Positive
Returns Possible

Subjective returns

\[ E(r) = \sum_{t=1}^{s} p(s)r(s) \]

\[ p(s) = \text{probability of a state} \]
\[ r(s) = \text{return if a state occurs} \]
1 to \( s \) states

Measuring Mean: Scenario or Subjective Returns

Numerical Example: Subjective or Scenario Distributions

<table>
<thead>
<tr>
<th>State</th>
<th>Prob. of State</th>
<th>( r_{in} )</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1</td>
<td>-.05</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>.2</td>
<td>.05</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>.4</td>
<td>.15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.2</td>
<td>.25</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.1</td>
<td>.35</td>
<td></td>
</tr>
</tbody>
</table>

\[ E(r) = (.1)(-.05) + (.2)(.05) + (.4)(.15) + (.2)(.25) + (.1)(.35) \]
\[ E(r) = .15 \text{ or } 15\% \]

Measuring Variance or Dispersion of Returns

\[ Var(r) = \sum_{t=1}^{s} p(s)[r(s) - E(r)]^2 \]
\[ SD(r) \equiv \sigma = \sqrt{Var(r)} \]
Measuring Variance or Dispersion of Returns

Using Our Example:

\[ \text{Var} = (0.1)(-0.15)^2 + (0.2)(-0.05)^2 + ... + (0.1)(-0.05)^2 \]
\[ \text{Var} = 0.01199 \]
\[ \text{S.D.} = \sqrt{0.01199} \approx 0.1095 \text{ or } 10.95\% \]

Risk Premiums and Risk Aversion

- Degree to which investors are willing to commit funds
  - Risk aversion
- If T-Bill denotes the risk-free rate, \( r_f \), and variance, \( \sigma^2 \), denotes volatility of returns then:
  \[ E(r_p) - r_f \]

The Sharpe (Reward-to-Volatility) Measure

\[ S = \frac{\text{portfolio risk premium}}{\text{standard deviation of portfolio excess return}} \]
\[ S = \frac{E(r_p) - r_f}{\sigma_p} \]

Annual Holding Period Returns

From Table 5.3 of Text

<table>
<thead>
<tr>
<th>Series</th>
<th>Geom. Mean%</th>
<th>Arith. Mean%</th>
<th>Stan. Dev.%</th>
</tr>
</thead>
<tbody>
<tr>
<td>World Stk</td>
<td>9.80</td>
<td>11.32</td>
<td>18.05</td>
</tr>
<tr>
<td>US Lg Stk</td>
<td>10.23</td>
<td>12.19</td>
<td>20.14</td>
</tr>
<tr>
<td>US Sm Stk</td>
<td>12.43</td>
<td>18.14</td>
<td>36.93</td>
</tr>
<tr>
<td>Wor Bonds</td>
<td>5.80</td>
<td>6.17</td>
<td>9.05</td>
</tr>
<tr>
<td>LT Treas.</td>
<td>5.35</td>
<td>5.64</td>
<td>8.06</td>
</tr>
<tr>
<td>T-Bills</td>
<td>3.72</td>
<td>3.77</td>
<td>3.11</td>
</tr>
<tr>
<td>Inflation</td>
<td>3.04</td>
<td>3.13</td>
<td>4.27</td>
</tr>
</tbody>
</table>
Annual Holding Period Excess Returns
From Table 5.3 of Text

<table>
<thead>
<tr>
<th>Series</th>
<th>Risk</th>
<th>Stan. Dev. %</th>
<th>Sharpe Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>World Stk</td>
<td>7.56</td>
<td>18.37</td>
<td>0.41</td>
</tr>
<tr>
<td>US Lg Stk</td>
<td>8.42</td>
<td>20.42</td>
<td>0.41</td>
</tr>
<tr>
<td>US Sm Stk</td>
<td>14.37</td>
<td>37.53</td>
<td>0.38</td>
</tr>
<tr>
<td>Wor Bonds</td>
<td>2.40</td>
<td>8.92</td>
<td>0.27</td>
</tr>
<tr>
<td>LT Treas</td>
<td>1.88</td>
<td>7.87</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 5.1 Frequency Distributions of Holding Period Returns

Figure 5.2 Rates of Return on Stocks, Bonds and T-Bills

Figure 5.3 Normal Distribution with Mean of 12% and St Dev of 20%

Table 5.4 Size-Decile Portfolios

<table>
<thead>
<tr>
<th>Decile</th>
<th>Geometric Average</th>
<th>Arithmetic Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Large</td>
<td>9.6%</td>
<td>11.4%</td>
<td>19.1%</td>
</tr>
<tr>
<td>2</td>
<td>10.9</td>
<td>13.2%</td>
<td>21.6%</td>
</tr>
<tr>
<td>3</td>
<td>11.4%</td>
<td>13.8%</td>
<td>22.9%</td>
</tr>
<tr>
<td>4</td>
<td>11.9%</td>
<td>14.4%</td>
<td>25.2%</td>
</tr>
<tr>
<td>5</td>
<td>12.0%</td>
<td>15.2%</td>
<td>26.6%</td>
</tr>
<tr>
<td>6</td>
<td>12.1%</td>
<td>15.6%</td>
<td>27.6%</td>
</tr>
<tr>
<td>7</td>
<td>12.4%</td>
<td>16.3%</td>
<td>30.0%</td>
</tr>
<tr>
<td>8</td>
<td>12.5%</td>
<td>17.0%</td>
<td>32.5%</td>
</tr>
<tr>
<td>9</td>
<td>12.2%</td>
<td>17.5%</td>
<td>35.3%</td>
</tr>
<tr>
<td>10 Smallest</td>
<td>13.8%</td>
<td>20.4%</td>
<td>49.9%</td>
</tr>
</tbody>
</table>

5.4 INFLATION AND REAL RATES OF RETURN
Real vs. Nominal Rates

Fisher effect: Approximation
nominal rate = real rate + inflation premium
R = r + i or R = r + i

Example: r = 3%, i = 6%
R = 9% = 3% + 6% or 3% = 9% - 6%

Real vs. Nominal Rates

Fisher effect:
R = r + i or R = r + i

Example: r = 3%, i = 6%
R = 9% = 3% + 6% or 3% = 9% - 6%

2.83% = (9% - 6%) / (1.06)

5.5 ASSET ALLOCATION ACROSS RISKY AND RISK-FREE PORTFOLIOS

Allocating Capital

- Possible to split investment funds between safe and risky assets
- Risk free asset: proxy; T-bills
- Risky asset: stock (or a portfolio)

Issues
- Examine risk/return tradeoff
- Demonstrate how different degrees of risk aversion will affect allocations between risky and risk free assets
The Risky Asset: Text Example (Page 143)

Total portfolio value = $300,000
Risk-free value = 90,000
Risky (Vanguard and Fidelity) = 210,000
Vanguard (V) = 54%
Fidelity (F) = 46%

Calculating the Expected Return Text Example (Page 145)

\[ r_f = 7\% \quad \sigma_{rf} = 0\% \]
\[ E(r_p) = 15\% \quad \sigma_p = 22\% \]
\[ y = \% \text{ in } p \quad (1-y) = \% \text{ in } r_f \]

Expected Returns for Combinations

\[ E(r_c) = yE(r_p) + (1-y)r_f \]
\[ r_c = \text{complete or combined portfolio} \]

For example, \( y = .75 \)
\[ E(r_c) = .75(.15) + .25(.07) = .13 \text{ or } 13\% \]

Figure 5.5 Investment Opportunity Set with a Risk-Free Investment

Since \( \sigma_{rf} = 0 \), then
\[ \sigma_c = y \sigma_p \]
Combinations Without Leverage

If \( y = .75 \), then
\[
\sigma_c = .75 \times (22) = .165 \text{ or } 16.5\% 
\]
If \( y = 1 \)
\[
\sigma_c = 1 \times (22) = .22 \text{ or } 22\% 
\]
If \( y = 0 \)
\[
\sigma_c = 0 \times (22) = .00 \text{ or } 0\% 
\]

Using Leverage with
Capital Allocation Line

Borrow at the Risk-Free Rate and invest in stock
Using 50% Leverage
\[
r_c = (-.5) \times (.07) + (1.5) \times (.15) = .19 
\]
\[
\sigma_c = (1.5) \times (22) = .33 
\]

Risk Aversion and Allocation

- Greater levels of risk aversion lead to larger proportions of the risk free rate
- Lower levels of risk aversion lead to larger proportions of the portfolio of risky assets
- Willingness to accept high levels of risk for high levels of returns would result in leveraged combinations

5.6 PASSIVE STRATEGIES AND THE CAPITAL MARKET LINE

Table 5.5 Average Rates of Return, Standard Deviation and Reward to Variability

<table>
<thead>
<tr>
<th>Year</th>
<th>Average</th>
<th>SD</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1926-1946</td>
<td>8.36</td>
<td>27.98</td>
<td>0.30</td>
</tr>
<tr>
<td>1947-1966</td>
<td>12.72</td>
<td>18.05</td>
<td>0.70</td>
</tr>
<tr>
<td>1967-1985</td>
<td>4.14</td>
<td>17.44</td>
<td>0.24</td>
</tr>
<tr>
<td>1986-2006</td>
<td>8.47</td>
<td>16.32</td>
<td>0.52</td>
</tr>
<tr>
<td>2006-2009</td>
<td>8.42</td>
<td>20.42</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Costs and Benefits of Passive Investing

- Active strategy entails costs
- Free-rider benefit
- Involves investment in two passive portfolios
  - Short-term T-bills
  - Fund of common stocks that mimics a broad market index