Chapter 4

Introduction to Valuation: The Time Value of Money

Key Concepts and Skills

- Be able to compute the future value of an investment made today
- Be able to compute the present value of cash to be received at some future date
- Be able to compute the return on an investment

Chapter Outline

- Future Value and Compounding
- Present Value and Discounting
- More on Present and Future Values

Basic Definitions

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – "exchange rate" between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

Future Values

- Suppose you invest $1,000 for one year at 5% per year. What is the future value in one year?
  - Interest = $1,000(.05) = $50
  - Value in one year = principal + interest = $1,000 + 50 = $1,050
  - Future Value (FV) = $1,000(1 + .05) = $1,050
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV = $1,000(1.05)(1.05) = $1,000(1.05)^2 = $1,102.50

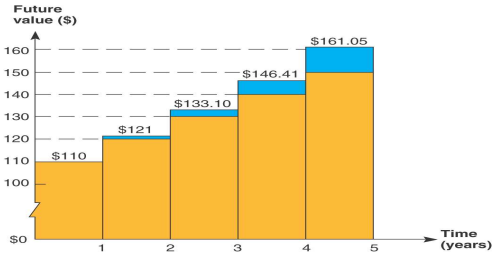
Future Values: General Formula

- FV = PV(1 + r)^t
  - FV = future value
  - PV = present value
  - r = period interest rate, expressed as a decimal
  - T = number of periods
- Future value interest factor = (1 + r)^t

Effects of Compounding

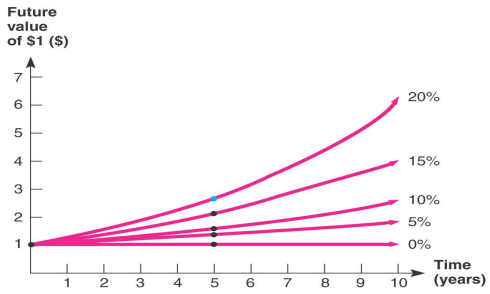
- Simple interest (interest is earned only on the original principal)
- Compound interest (interest is earned on principal and on interest received)
- Consider the previous example
  - FV with simple interest = $1,000 + 50 + 50 = $1,100
  - FV with compound interest = $1,102.50
  - The extra $2.50 comes from the interest of .05($50) = $2.50 earned on the first interest payment

Figure 4.1



Growth of $100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

Figure 4.2



Calculator Keys

- Texas Instruments BA-II Plus
  - FV = future value
  - PV = present value
  - I/Y = period interest rate
    - P/Y must equal 1 for the I/Y to be the period rate
    - Interest is entered as a percent, not a decimal
  - N = number of periods
  - Remember to clear the registers (CLR TVM) before (and after) each problem
  - Other calculators are similar in format

Future Values – Example 2

- Suppose you invest the $1,000 from the previous example for 5 years. How much would you have?
  - FV = $1,000(1.05)^5 = $1,276.28
- The effect of compounding is small for a small number of periods, but increases as the number of periods increases. (Simple interest would have a future value of $1,250, for a difference of $26.28.)

Future Values – Example 3

- Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - FV = $10(1.055)^{200} = $447,189.84
- What is the effect of compounding?
  - Simple interest = $10 + $10(200)(.055) = $120
  - Compounding added $447,069.84 to the value of the investment

Future Value as a General Growth Formula

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell during the fifth year?
  - FV = $3{,}000{,}000(1.15)^5 = 6{,}034{,}072$

Quick Quiz: Part 1

- What is the difference between simple interest and compound interest?
- Suppose you have $500 to invest and you believe that you can earn 8% per year over the next 15 years.
  - How much would you have at the end of 15 years using compound interest?
  - How much would you have using simple interest?

Present Values

- How much do I have to invest today to have some amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the "value" of something, we are talking about the present value unless we specifically indicate that we want the future value.

PV – One-Period Example

- Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?
- $PV = \$10{,}000 / (1.07)^1 = \$9{,}345.79$
- Calculator
  - 1 N
  - 7 I/Y
  - 10,000 FV
  - CPT PV = -9,345.79

Present Values – Example 2

- You want to begin saving for your daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
  - $PV = \$150{,}000 / (1.08)^{17} = \$40{,}540.34$

Present Values – Example 3

- Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?
  - $PV = \$19{,}671.51 / (1.07)^{10} = \$10{,}000$

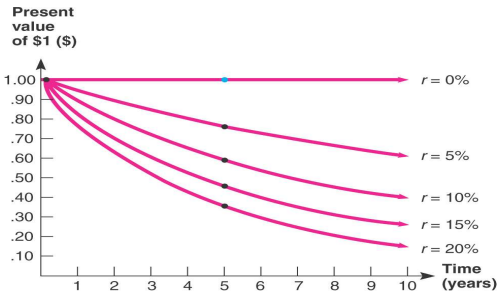
PV – Important Relationship I

- For a given interest rate – the longer the time period, the lower the present value (ceteris paribus: all else equal)
  - What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%
  - 5 years: PV = $500 / $(1.1)^5$ = $310.46
  - 10 years: PV = $500 / $(1.1)^{10}$ = $192.77

PV – Important Relationship II

- For a given time period – the higher the interest rate, the smaller the present value (ceteris paribus)
  - What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%: PV = $500 / $(1.1)^5$ = $310.46
    - Rate = 15%; PV = $500 / $(1.15)^5$ = $248.59

Figure 4.3



Quick Quiz: Part 2

- What is the relationship between present value and future value?
- Suppose you need $15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?
- If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

The Basic PV Equation - Refresher

- PV = FV / $(1 + r)^t$
- There are four parts to this equation
  - PV, FV, r, and t
  - If we know any three, we can solve for the fourth
- If you are using a financial calculator, be sure to remember the sign convention or you will receive an error when solving for r or t

Discount Rate

- Often, we will want to know what the implied interest rate is in an investment
- Rearrange the basic PV equation and solve for r
  - FV = $PV(1 + r)^t$
  - r = $(FV / PV)^{1/t} – 1$
- If you are using formulas, you will want to make use of both the y^x and the 1/x keys

Discount Rate – Example 1

- You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. What is the implied rate of interest?
  - r = ($1,200 / $1,000)$^{1/5}$ – 1 = .03714 = 3.714%
  - Calculator – the sign convention matters!!!
    - N = 5
    - PV = -1,000 (you pay $1,000 today)
    - FV = 1,200 (you receive $1,200 in 5 years)
    - CPT I/Y = 3.714%

Discount Rate – Example 2

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have $10,000 to invest. What is the implied rate of interest?
  - r = ($20,000 / $10,000)$^{1/6}$ – 1 = .122462 = 12.25%

Discount Rate – Example 3

- Suppose you have a 1-year old son and you want to provide $75,000 in 17 years toward his college education. You currently have $5,000 to invest. What interest rate must you earn to have the $75,000 when you need it?
  - r = ($75,000 / $5,000)$^{1/17}$ – 1 = .172686 = 17.27%

Quick Quiz: Part 3

- What are some situations in which you might want to compute the implied interest rate?
- Suppose you are offered the following investment choices:
  - You can invest $500 today and receive $600 in 5 years. The investment is considered low risk.
  - You can invest the $500 in a bank account paying 4% annually.
  - What is the implied interest rate for the first choice and which investment should you choose?

Finding the Number of Periods

- Start with basic equation and solve for t (remember your logs)
  - FV = PV(1 + r)^t
  - t = ln(FV / PV) / ln(1 + r)
- You can use the financial keys on the calculator as well. Just remember the sign convention.

Number of Periods – Example 1

- You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?
  - t = ln($20,000 / $15,000) / ln(1.1) = 3.02 years

Number of Periods – Example 2

- Suppose you want to buy a new house. You currently have $15,000 and you figure you need to have a 10% down payment plus an additional 5% in closing costs. If the type of house you want costs about $150,000 and you can earn 7.5% per year, how long will it be before you have enough money for the down payment and closing costs?

Example 2 Continued

- How much do you need to have in the future?
  - Down payment = .1($150,000) = $15,000
  - Closing costs = .05($150,000 – 15,000) = $6,750
  - Total needed = $15,000 + 6,750 = $21,750
- Compute the number of periods
  - PV = -15,000
  - FV = 21,750
  - I/Y = 7.5
  - CPT N = 5.14 years
- Using the formula
  - t = ln($21,750 / $15,000) / ln(1.075) = 5.14 years

Table 4.4

I. **Symbols**
PV = Present value, what future cash flows are worth today
FV_t = Future value, what cash flows are worth in the future
r = Interest rate, rate of return, or discount rate per period typically , but not always, one year
t = Number of periods typically , but not always, the number of years
C = Cash amount

II. **Future value of *C* invested at *r* percent per period for *t* periods**
$FV_t = C \times (1 + r)^t$
The term $(1 + r)^t$ is called the *future value factor.*

III. **Present value of *C* to be received in *t* periods at *r* percent per period**
$PV = C/(1 + r)^t$
The term $1/(1 + r)^t$ is called the *present value factor.*

IV. **The basic present value equation giving the relationship between present and future value is**
$PV = FV_t/(1 + r)^t$

Quick Quiz: Part 4

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room. You currently have $500 and the furniture you want costs $600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

Comprehensive Problem

- You have $10,000 to invest for five years.
- How much additional interest will you earn if the investment provides a 5% annual return, when compared to a 4.5% annual return?
- How long will it take your $10,000 to double in value if it earns 5% annually?
- What annual rate has been earned if $1,000 grows into $4,000 in 20 years?