Probability Distribution Applications and Relationships

from *Statistical Models in Engineering*, by Gerald Hahn and Samuel Shapiro, John Wiley & Sons, © 1967, pages 133, 134 and 163

Table 4-3 Summary: Applications of Discrete Statistical Distributions

Distribution	Application	Example	Comments
Binomial	Gives probability of exactly x successes in n independent trials, when probability of success p on *single* trial is a constant. Used frequently in quality control, reliability, survey sampling, and other industrial problems.	What is the probability of 7 or more "heads" in 10 tosses of a fair coin?	Can sometimes be approximated by normal or by Poisson distribution.
Multinomial	Gives probability of exactly x_i outcomes of event i, for $i = 1, 2, \ldots, k$ in n independent trials when the probability p_i of event i in a *single* trial is a constant. Used frequently in quality control and other industrial problems.	Four companies are bidding for each of three contracts, with specified success probabilities. What is the probability that a single company will receive all the orders?	Generalization of binomial distribution for more than 2 outcomes.
Hypergeometric	Gives probability of picking exactly x good units in a sample of n units from a population of N units when there are k bad units in the population. Used in quality control and related applications.	Given a lot with 21 good units and four defectives. What is the probability that a sample of five will yield not more than one defective?	May be approximated by binomial distribution when n is small relative to N.
Geometric	Gives probability of requiring exactly x binomial trials before the first success is achieved. Used in quality control, reliability, and other industrial situations.	Determination of probability of requiring exactly five test firings before first success is achieved.	
Pascal	Gives probability of exactly x failures preceding the sth success.	What is the probability that the third success takes place on the 10th trial?	
Negative Binomial	Gives probability similar to Poisson distribution (see below) when events do not occur at a constant rate and occurrence rate is a random variable that follows a gamma distribution.	Distribution of number of cavities for a group of dental patients.	Generalization of Pascal distribution when s is not an integer. Many authors do not distinguish between Pascal and negative binomial distributions.
Poisson	Gives probability of exactly x independent occurrences during a given period of time if events take place independently and at a constant rate. May also represent number of occurrences over constant areas or volumes. Used frequently in quality control, reliability, queueing theory, and so on.	Used to represent distribution of number of defects in a piece of material, customer arrivals, insurance claims, incoming telephone calls, alpha particles emitted, and so on.	Frequently used as approximation to binomial distribution.

Table 3-2 Summary: Applications of Continuous Statistical Distributions

Distribution	Application	Example	Comments
Normal	A basic distribution of statistics. Many applications arise from central limit theorem (average of values of n observations approaches normal distribution, irrespective of form of original distribution under quite general conditions). Consequently, appropriate model for many—but not all—physical phenomena.	Distribution of physical measurements on living organisms, intelligence test scores, product dimensions, average temperatures, and so on.	Tabulation of cumulative values of standardized normal distribution readily available. Many methods of statistical analysis presume normal distribution.
Gamma	A basic distribution of statistics for variables bounded at one side—for example, $0 \leq x < \infty$. Gives distribution of time required for exactly k independent events to occur, assuming events take place at a constant rate. Used frequently in queueing theory, reliability, and other industrial applications.	Distribution of time between recalibrations of instrument that needs recalibration after k uses; time between inventory restocking, time to failure for a system with standby components.	Cumulative distribution values have been tabulated. Erlangian, exponential, and chi-square distributions are special cases.
Exponential	Gives distribution of time between independent events occurring at a constant rate. Equivalently, probability distribution of life, presuming constant conditional failure (or hazard) rate. Consequently, applicable in many—but not all—reliability situations.	Distribution of time between arrival of particles at a counter. Also life distribution of complex nonredundant systems, and usage life of some components—in particular, when these are exposed to initial burn-in, and preventive maintenance eliminates parts before wear-out.	Special case of both Weibull and gamma distributions.
Beta	A basic distribution of statistics for variables bounded at both sides—for example $0 \leq x \leq 1$. Useful for both theoretical and applied problems in many areas.	Distribution of proportion of population located between lowest and highest value in sample; distribution of daily per cent yield in a manufacturing process; description of elapsed times to task completion (PERT).	Cumulative distribution values have been tabulated. Uniform, right triangular, and parabolic distributions are special cases.

Table 3-2 (continued)

Summary: Applications of Continous Statistical Distributions

Distribution	Application	Example	Comments
Uniform	Gives probability that observation will occur within a particular interval when probability of occurrence within that interval is directly proportional to interval length.	Used to generate random values.	Special case of beta distribution.
Log-normal	Permits representation of random variable whose logarithm follows normal distribution. Model for a process arising from many small multiplicative errors. Appropriate when the value of an observed variable is a random proportion of the previously observed value.	Distribution of sizes from a breakage process; distribution of income size, inheritances and bank deposits; distribution of various biological phenomena; life distribution of some transistor types.	
Rayleigh	Gives distribution of radial error when the errors in two mutually perpendicular axes are independent and normally distributed around zero with equal variances.	Bomb-sighting problems; amplitude of noise envelope when a linear detector is used.	Special case of Weibull distribution.
Cauchy	Gives distribution of ratio of two independent standardized normal variates.	Distribution of ratio of standardized noise readings; distribution of tan θ when θ is uniformly distributed.	Has no moments.
Weibull	General time-to-failure distribution due to wide diversity of hazard-rate curves, and extreme-value distribution for minimum of N values from distribution bounded at left.	Life distribution for some capacitors, ball bearings, relays, and so on.	Rayleigh and exponential distributions are special cases.
Extreme value	Limiting model for the distribution of the maximum or minimum of N values selected from an "exponential-type" distribution, such as the normal, gamma, or exponential.	Distribution of breaking strength of some materials, capacitor breakdown voltage, gust velocities encountered by airplanes, bacteria extinction times.	Cumulative distribution has been tabulated.