

PROBABILITY DISTRIBUTIONS

**For basic probability concepts,
had “simple” random experiments
with some specific outcomes and numerical values**

**Now, define a *Random Variable* as a *function*
that assigns a real number to each outcome
in the sample space of a random experiment**

Further, define

Discrete Random Variable

and

Continuous Random Variable

Discrete Random Variable

**A random variable with a finite (or countably infinite) range
(i.e., the set of possible values of a random variable X)**

Examples of discrete random variables:

Number of scratches on a surface

Number of transmitted bits received in error

Proportion of defective parts among 1,000 tested

Distributions of discrete random variables include:

Binomial

Multinomial

Hypergeometric

Poisson

Geometric

Pascal

Negative Binomial

Continuous Random Variable

A random variable with an interval (either finite or infinite) of real numbers for its range

Examples of continuous random variables:

Length

Pressure

Height

Time

Distributions of continuous random variables include:

Normal

Gamma

Exponential

Beta

Uniform

Log-normal

Rayleigh

Cauchy

Weibull

Extreme value

BINOMIAL DISTRIBUTION

- Independent repeated trials (known as Bernoulli trials)
- Each trial results in one of two possible outcomes (i.e., “success” or “failure”)
- Probability of “success,” p , remains constant from one trial to the next (i.e., “sampling with replacement”)

EXAMPLE APPLICATIONS

- Ten space shots are planned. Each has a probability of success of 0.95. What are the chances that at least nine shots will be successful?
- Items are manufactured in large lots, from each of which twenty units are selected at random. The lot is accepted if the sample contains three or fewer defective units. If the production process yields on the average 10 per cent defectives, what is the probability of lot acceptance?

BINOMIAL DISTRIBUTION

Binomial Probability Function:

$$f(x; p, n) = \binom{n}{x} p^x (1-p)^{n-x},$$

$$x = 0, 1, 2, \dots, n; \quad 0 \leq p \leq 1$$

where n is the number of trials

x is the number of successes in
 n trials

p is the probability of success
in a single trial

Note that $\binom{n}{x} = nCx = \frac{n!}{x!(n-x)!}$

Note: $(1-p)$ is often referred to as q .

BINOMIAL DISTRIBUTION

Distribution Function:

The probability of x or fewer successes in n independent trials, given a probability of success p in a single trial:

$$\begin{aligned} \Pr(X \leq x) &= F(x; p, n) \\ &= \sum_{k=0}^x \binom{n}{k} p^k (1-p)^{n-k} \end{aligned}$$

$$F(x) = 0 \text{ when } x < 0$$

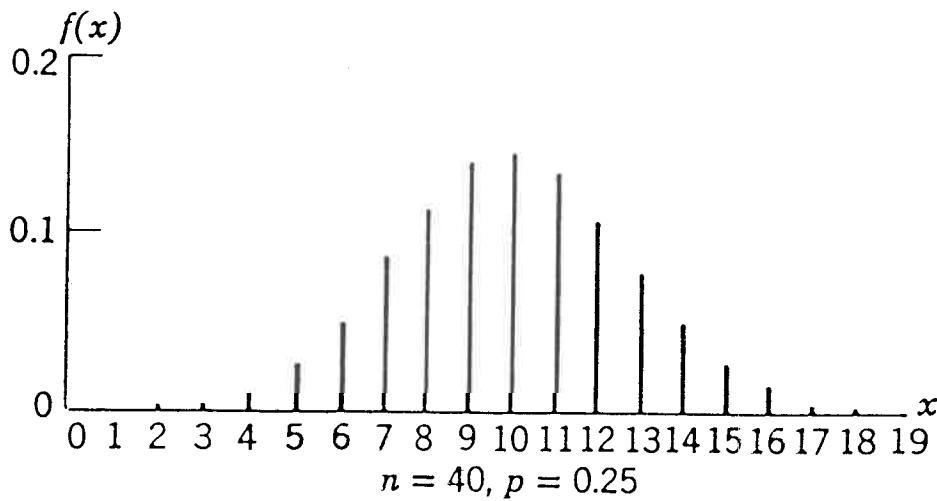
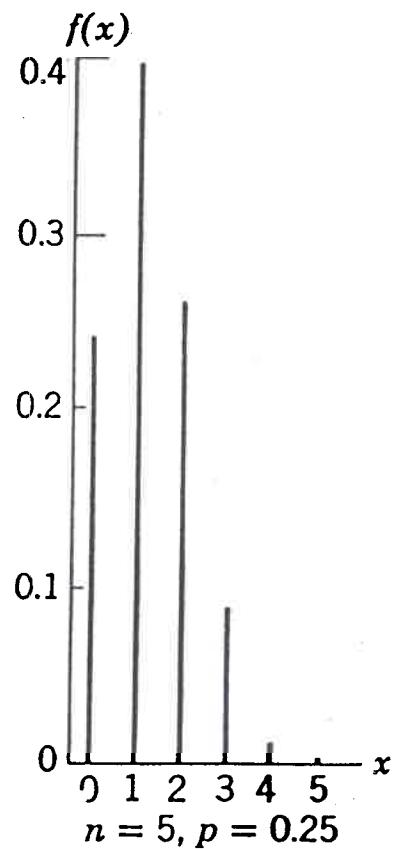
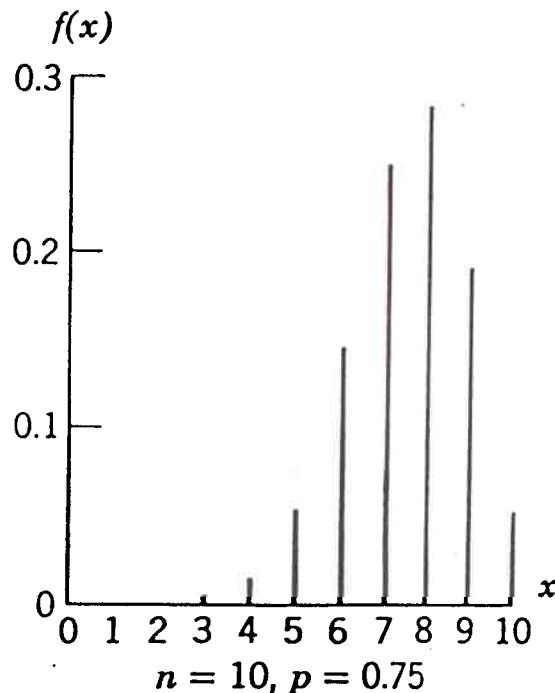
Mean and Variance of Binomial Distribution:

$$\text{mean} = \mu = np$$

$$\begin{aligned} \text{variance} &= \sigma^2 = np(1-p) \\ &= npq, \text{ where } q = (1-p) \end{aligned}$$

BINOMIAL DISTRIBUTION

Example Probability Functions:



BINOMIAL DISTRIBUTION

Tabled Values:



Table 1 Binomial Distribution Function

$$B(x; n, p) = \sum_{k=0}^x \binom{n}{k} p^k (1-p)^{n-k}$$

n	x	p						
		0.05	0.10	0.15	0.20	0.25	0.30	0.35
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2740
	1	0.9927	0.9720	0.9393	0.8960	0.8438	0.7840	0.7040
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9500
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1701
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5555
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9162	0.8648
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9916	0.9856
5	0	0.7738	0.5905	0.4437	0.3164	0.2143	0.1335	0.0843
	1	0.9774	0.9185	0.8225	0.7049	0.5625	0.4096	0.2740
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9500

BINOMIAL DISTRIBUTION

Example Problem: (*Source: Montgomery & Runger*)

Samples of air each have a 10% chance of containing a particular molecule. Assume that the samples are independent with regard to the presence of the molecule. Find the probability that:

- (a) Of 18 samples, exactly 2 contain the molecule, and
- (b) Of the 18 samples, at least 4 contain the molecule.

Let $x \Rightarrow$ contains the molecule

Given $p = 0.10$ and $n = 18$

$$\begin{aligned}
 (a) \Pr(X=2) &= \binom{18}{2} (0.1)^2 (0.9)^{16} \\
 &= \frac{18!}{2! 16!} (0.1)^2 (0.9)^{16} \\
 &\doteq 0.284 \text{ or } 28.4\%
 \end{aligned}$$

$$\begin{aligned}
 (b) \Pr(X \geq 4) &= \underline{\Pr(X \leq 3)} \\
 \Pr(X \leq 3) &= 0.9018
 \end{aligned}$$

$$\begin{aligned}
 \text{Thus } \Pr(X \geq 4) &= 1 - 0.9018 \\
 &\doteq 0.0982 \text{ or } 9.8\%
 \end{aligned}$$

HYPERGEOMETRIC DISTRIBUTION

- Probability of exactly x successes in n trials, when n is not small relative to the population size
- Frequently applicable to problems involving sampling from a small lot (often referred to as “sampling without replacement”)

EXAMPLE APPLICATIONS

- From an order for 25 high-reliability items, five are selected at random and life tested. If fewer than two items fail this test, the remaining 20 items will be accepted. Otherwise, the complete lot will be rejected. What is the probability of lot acceptance if four of 25 submitted items are defective?
- A radio manufacturing company puts into a container 24 items for shipment. An inspector at the plant will test two items at random prior to shipping, and if any of the tested items prove defective he will return the whole container for examination. If there are actually two defectives in each container, what is the probability that a box will not be shipped?

HYPERGEOMETRIC DISTRIBUTION

Probability Function:

Probability of exactly x defective units in randomly selecting n units from a lot of N units, of which exactly k are defective

$$f(x) \\ f(x; N, n, k) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}}$$

$$x = 0, 1, 2, \dots, n ; \quad x \leq k \\ (n-x) \leq (N-k)$$

Mean and Variance of Hypergeometric Distribution:

$$\text{mean} = \mu = \frac{nk}{N}$$

$$\text{variance} = \sigma^2 = \frac{nk(N-k)(N-n)}{N^2(N-1)}$$

POISSON DISTRIBUTION

- **Probability of exactly x independent occurrences in a given time period (or volume or area)**
- **Assumes occurrences take place independently and at a constant rate**

EXAMPLE APPLICATIONS

- **In 10 random commuter train trips from Santa Barbara to Los Angeles, a total of 15 separate and independent delays were recorded. The train company claims that the long-run average number of delays per run is 1.2. If the number of delays is a Poisson variate, do the data refute the company's claim?**
- **The average number of independent insulation breaks per yard in an enameling process is 0.07. What is the probability of finding one break in a piece of wire 16 yards long?**

POISSON DISTRIBUTION

Probability Function:

$$f(x; \lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$x = 0, 1, 2, \dots ; \lambda > 0$$

where λ = mean rate of occurrence

Distribution Function:

$$F(x) = e^{-\lambda} \sum_{s \leq x} \frac{\lambda^s}{s!}$$

$$F(x) = 0 \text{ when } x < 0$$

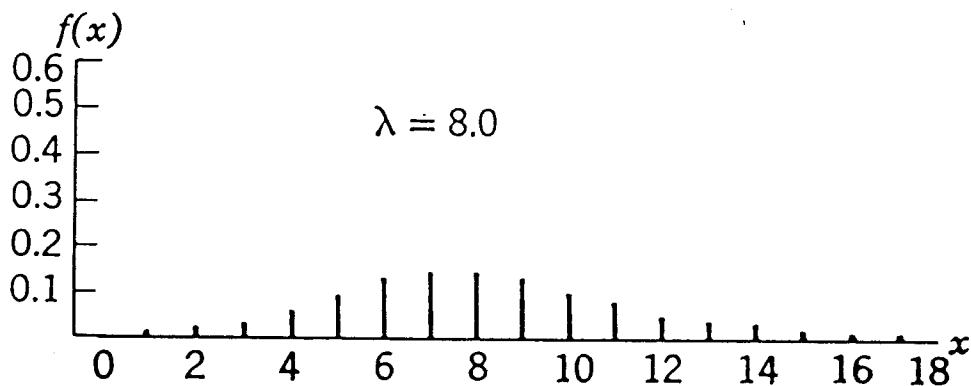
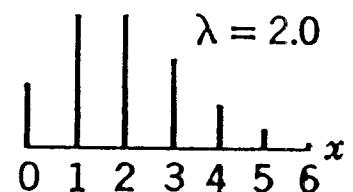
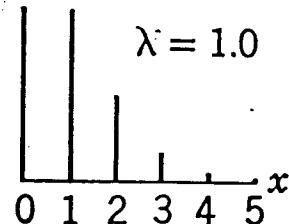
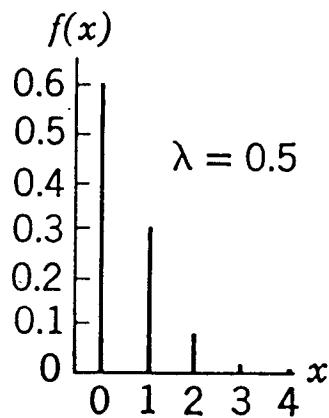
Mean and Variance of Poisson Distribution:

$$\text{mean} = \mu = \lambda$$

$$\text{variance} = \sigma^2 = \lambda$$

POISSON DISTRIBUTION

Example Probability Functions:



POISSON DISTRIBUTION

Tabled Values:

Table 2 Poisson Distribution Function

$$F(x; \lambda) = \sum_{k=0}^x e^{-\lambda} \frac{\lambda^k}{k!}$$

$\lambda \backslash x$	0	1	2	3	
0.02	0.980	1.000			
0.04	0.961	0.999	1.000		
0.06	0.942	0.998	1.000		
0.08	0.923	0.997	1.000		
0.10	0.905	0.995	1.000		
0.15	0.861	0.990	0.999	1.000	
0.20	0.819	0.982	0.999	1.000	
0.25	0.779	0.974	0.998	1.000	
0.30	0.741	0.963	0.996		
0.35	0.705				
0.40					

POISSON DISTRIBUTION

Example Problem: (source: Montgomery + Runger)

Contamination is a problem in the manufacture of optical storage disks. The number of particles of contamination that occur on an optical disk has a Poisson distribution, and the average number of particles per centimeter squared of media surface is 0.1. The area of a disk under study is 100 square centimeters. Find the probability that:

- (a) 12 particles occur in the area of this disk
- (b) no more than 12 particles occur in the area of this disk

Let $x \Rightarrow$ particles in the area of the disk under study

$$\lambda = (100 \text{ cm}^2)(0.1 \text{ particles/cm}^2) = 10 \text{ particles/disk}$$

*Indiv
Prob
function* (a) $\Pr(X=12) = \frac{e^{-10} 10^{12}}{12!} \doteq 0.095$ (*using calculator*)

*Cum
dist.
function* (b) $\Pr(X \leq 12) \doteq 0.792$

Note: part (a) can be solved using tabled values by realizing

$$\begin{aligned}\Pr(X=12) &= \Pr(X \leq 12) - \Pr(X \leq 11) \\ &= 0.792 - 0.697 \\ &= 0.095\end{aligned}$$

GEOMETRIC DISTRIBUTION

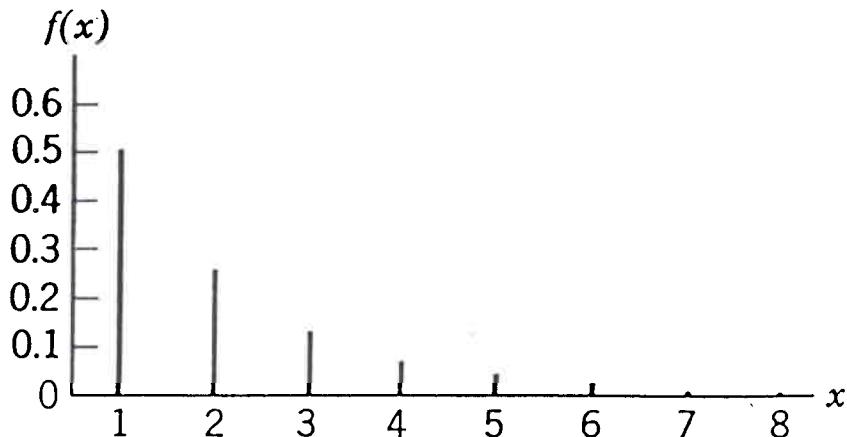
Probability Function:

$$f(x; p) = p (1-p)^{x-1}$$

$$x = 1, 2, 3, \dots ; \quad 0 \leq p \leq 1$$

where x is the number of trials
 (until the first success)
 p is the probability of "success"

Example Probability Function:



Geometric distribution with $p = \frac{1}{2}$

Mean and Variance of Geometric Distribution:

$$\text{mean} = \mu = \frac{1}{p}$$

$$\text{variance} = \sigma^2 = \frac{(1-p)}{p^2}$$

GEOMETRIC DISTRIBUTION

- Appropriate to problems in which we desire the probability that exactly x independent Bernoulli trials, each with a probability of “success,” p , will be required until the first “success” is achieved
- Special case of the Pascal distribution, which is a special case of the generalized negative binomial distribution

EXAMPLE APPLICATION

- The probability is 0.1 that a bit transmitted through a digital transmission channel is received in error. Assume the transmissions are independent events, and that the number of bits transmitted until the first error is a random variable. What is the probability that the first incorrect bit will be the fifth one?

MSE 362
By Campbell

Table 1 Binomial Distribution Function (5 pages)
 $B(x; n, p) = \sum_{k=0}^x \binom{n}{k} p^k (1-p)^{n-k}$

n	x	p																		
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0.2025	0.1600	0.1225	0.0900	0.0625	0.0400	0.0225	0.0100	0.0025
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	0.6975	0.6400	0.5775..	0.5100	0.4375	0.3600	0.2775	0.1900	0.0975
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0.0911..	0.0640	0.0429	0.0270	0.0156	0.0080	0.0034	0.0010	0.0001
	1	0.9927	0.9720	0.9393	0.8950	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	0.4252	0.3520	0.2818	0.2160	0.1563	0.1040	0.0607	0.0280	0.0073
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0.0410	0.0256	0.0150	0.0081	0.0039	0.0016	0.0005	0.0001	0.0000
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	0.2415	0.1792	0.1265	0.0837	0.0508	0.0272	0.0120	0.0037	0.0005
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0024	0.0010	0.0003	0.0001	0.0000	0.0000
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0308	0.0156	0.0067	0.0022	0.0005	0.0000
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
	1	0.9672	0.8837	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0109	0.0046	0.0016	0.0004	0.0001	0.0000
7	0	0.6983	0.4783	0.3026	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0038	0.0013	0.0004	0.0001	0.0000	0.0000
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000

source: Johnson, Richard A. Miller and Freund's Probability and Statistics for Engineers. Prentice-Hall, 1994

(1)

n	x	p								
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046
1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9888	0.9750	0.9502
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805	0.9615
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025
1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
6	1.0000	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.992	0.9877	0.9726	0.9453
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9935
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014
1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0632	0.0327
3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
4	0.9999	0.9972	0.9841	0.9496	0.8834	0.7897	0.6683	0.5328	0.3971	0.2744
5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
6	1.0000	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262
7	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
8	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9832	0.9673	0.9348
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.9804	0.8891	0.7338	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.9998	0.9357	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
6	1.0000	1.0000	0.9999	0.9993	0.9961	0.9837	0.9614	0.9154	0.8418	0.7393
7	1.0000	1.0000	1.0000	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

2

n	x	P																		
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
11	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9988	0.9987	0.9986	0.9985	0.9984	0.9983
12	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9988	0.9987	0.9986	0.9985	0.9984	0.9983
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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Table 2 Poisson Distribution Function* (5 pages)

$$F(x; \lambda) = \sum_{k=0}^x e^{-\lambda} \frac{\lambda^k}{k!}$$

$\lambda \backslash x$	0	1	2	3	4	5	6	7	8	9
0.02	0.980	1.000								
0.04	0.961	0.999	1.000							
0.06	0.942	0.998	1.000							
0.08	0.923	0.997	1.000							
0.10	0.905	0.995	1.000							
0.15	0.861	0.990	0.999	1.000						
0.20	0.819	0.982	0.999	1.000						
0.25	0.779	0.974	0.998	1.000						
0.30	0.741	0.963	0.996	1.000						
0.35	0.705	0.951	0.994	1.000						
0.40	0.670	0.938	0.992	0.999	1.000					
0.45	0.638	0.925	0.989	0.999	1.000					
0.50	0.607	0.910	0.986	0.998	1.000					
0.55	0.577	0.894	0.982	0.998	1.000					
0.60	0.549	0.878	0.977	0.997	1.000					
0.65	0.522	0.861	0.972	0.996	0.999	1.000				
0.70	0.497	0.844	0.966	0.994	0.999	1.000				
0.75	0.472	0.827	0.959	0.993	0.999	1.000				
0.80	0.449	0.809	0.953	0.991	0.999	1.000				
0.85	0.427	0.791	0.945	0.989	0.998	1.000				
0.90	0.407	0.772	0.937	0.987	0.998	1.000				
0.95	0.387	0.754	0.929	0.984	0.997	1.000				
1.00	0.368	0.736	0.920	0.981	0.996	0.999	1.000			
1.1	0.333	0.699	0.900	0.974	0.995	0.999	1.000			
1.2	0.301	0.663	0.879	0.966	0.992	0.998	1.000			
1.3	0.273	0.627	0.857	0.957	0.989	0.998	1.000			
1.4	0.247	0.592	0.833	0.946	0.986	0.997	0.999	1.000		
1.5	0.223	0.558	0.809	0.934	0.981	0.996	0.999	1.000		
1.6	0.202	0.525	0.783	0.921	0.976	0.994	0.999	1.000		
1.7	0.183	0.493	0.757	0.907	0.970	0.992	0.998	1.000		
1.8	0.165	0.463	0.731	0.891	0.964	0.990	0.997	0.999	1.000	
1.9	0.150	0.434	0.704	0.875	0.956	0.987	0.997	0.999	1.000	
2.0	0.135	0.406	0.677	0.857	0.947	0.983	0.995	0.999	1.000	

* Reprinted by kind permission from E. C. Molina, Poisson's Exponential Binomial Limit, D. Van Nostrand Company, Inc., Princeton, N.J., 1947.

Source: Johnson, Richard A. Miller and Freund's Probability and Statistics for Engineers. Prentice-Hall, 1994.

Table 2 Poisson Distribution Function (Continued)

$\lambda \backslash x$	0	1	2	3	4	5	6	7	8	9
2.2	0.111	0.355	0.623	0.819	0.928	0.975	0.993	0.998	1.000	
2.4	0.091	0.308	0.570	0.779	0.904	0.964	0.988	0.997	0.999	1.000
2.6	0.074	0.267	0.518	0.736	0.877	0.951	0.983	0.995	0.999	1.000
2.8	0.061	0.231	0.469	0.692	0.848	0.935	0.976	0.992	0.998	0.999
3.0	0.050	0.199	0.423	0.647	0.815	0.916	0.966	0.988	0.996	0.999
3.2	0.041	0.171	0.380	0.603	0.781	0.895	0.955	0.983	0.994	0.998
3.4	0.033	0.147	0.340	0.558	0.744	0.871	0.942	0.977	0.992	0.997
3.6	0.027	0.126	0.303	0.515	0.706	0.844	0.927	0.969	0.988	0.996
3.8	0.022	0.107	0.269	0.473	0.668	0.816	0.909	0.960	0.984	0.994
4.0	0.018	0.092	0.238	0.433	0.629	0.785	0.889	0.949	0.979	0.992
4.2	0.015	0.078	0.210	0.395	0.590	0.753	0.867	0.936	0.972	0.989
4.4	0.012	0.066	0.185	0.359	0.551	0.720	0.844	0.921	0.964	0.985
4.6	0.010	0.056	0.163	0.326	0.513	0.686	0.818	0.905	0.955	0.980
4.8	0.008	0.048	0.143	0.294	0.476	0.651	0.791	0.887	0.944	0.975
5.0	0.007	0.040	0.125	0.265	0.440	0.616	0.762	0.867	0.932	0.968
5.2	0.006	0.034	0.109	0.238	0.406	0.581	0.732	0.845	0.918	0.960
5.4	0.005	0.029	0.095	0.213	0.373	0.546	0.702	0.822	0.903	0.951
5.6	0.004	0.024	0.082	0.191	0.342	0.512	0.670	0.797	0.886	0.941
5.8	0.003	0.021	0.072	0.170	0.313	0.478	0.638	0.771	0.867	0.929
6.0	0.002	0.017	0.062	0.151	0.285	0.446	0.606	0.744	0.847	0.916
	10	11	12	13	14	15	16			
2.8	1.000									
3.0	1.000									
3.2	1.000									
3.4	0.999	1.000								
3.6	0.999	1.000								
3.8	0.998	0.999	1.000							
4.0	0.997	0.999	1.000							
4.2	0.996	0.999	1.000							
4.4	0.994	0.998	0.999	1.000						
4.6	0.992	0.997	0.999	1.000						
4.8	0.990	0.996	0.999	1.000						
5.0	0.986	0.995	0.998	0.999	1.000					
5.2	0.982	0.993	0.997	0.999	1.000					
5.4	0.977	0.990	0.996	0.999	1.000					
5.6	0.972	0.988	0.995	0.998	0.999	1.000				
5.8	0.965	0.984	0.993	0.997	0.999	1.000				
6.0	0.957	0.980	0.991	0.996	0.999	0.999	1.000			

Table 2 Poisson Distribution Function (Continued)

$\lambda \backslash x$	0	1	2	3	4	5	6	7	8	9	
6.2	0.002	0.015	0.054	0.134	0.259	0.414	0.574	0.716	0.826	0.902	
6.4	0.002	0.012	0.046	0.119	0.235	0.384	0.542	0.687	0.803	0.886	
6.6	0.001	0.010	0.040	0.105	0.213	0.355	0.511	0.658	0.780	0.869	
6.8	0.001	0.009	0.034	0.093	0.192	0.327	0.480	0.628	0.755	0.850	
7.0	0.001	0.007	0.030	0.082	0.173	0.301	0.450	0.599	0.729	0.830	
7.2	0.001	0.006	0.025	0.072	0.156	0.276	0.420	0.569	0.703	0.810	
7.4	0.001	0.005	0.022	0.063	0.140	0.253	0.392	0.539	0.676	0.788	
7.6	0.001	0.004	0.019	0.055	0.125	0.231	0.365	0.510	0.648	0.765	
7.8	0.000	0.004	0.016	0.048	0.112	0.210	0.338	0.481	0.620	0.741	
8.0	0.000	0.003	0.014	0.042	0.100	0.191	0.313	0.453	0.593	0.717	
8.5	0.000	0.002	0.009	0.030	0.074	0.150	0.256	0.386	0.523	0.653	
9.0	0.000	0.001	0.006	0.021	0.055	0.116	0.207	0.324	0.456	0.587	
9.5	0.000	0.001	0.004	0.015	0.040	0.089	0.165	0.269	0.392	0.522	
10.0	0.000	0.000	0.003	0.010	0.029	0.067	0.130	0.220	0.333	0.458	
	10	11	12	13	14	15	16	17	18	19	
6.2	0.949	0.975	0.989	0.995	0.998	0.999	1.000				
6.4	0.939	0.969	0.986	0.994	0.997	0.999	1.000				
6.6	0.927	0.963	0.982	0.992	0.997	0.999	0.999	1.000			
6.8	0.915	0.955	0.978	0.990	0.996	0.998	0.999	1.000			
7.0	0.901	0.947	0.973	0.987	0.994	0.998	0.999	1.000			
7.2	0.887	0.937	0.967	0.984	0.993	0.997	0.999	0.999	1.000		
7.4	0.871	0.926	0.961	0.980	0.991	0.996	0.998	0.999	1.000		
7.6	0.854	0.915	0.954	0.976	0.989	0.995	0.998	0.999	1.000		
7.8	0.835	0.902	0.945	0.971	0.986	0.993	0.997	0.999	1.000		
8.0	0.816	0.888	0.936	0.966	0.983	0.992	0.996	0.998	0.999	1.000	
8.5	0.763	0.849	0.909	0.949	0.973	0.986	0.993	0.997	0.999	0.999	
9.0	0.706	0.803	0.876	0.926	0.959	0.978	0.989	0.995	0.998	0.999	
9.5	0.645	0.752	0.836	0.898	0.940	0.967	0.982	0.991	0.996	0.998	
10.0	0.583	0.697	0.792	0.864	0.917	0.951	0.973	0.986	0.993	0.997	
	20	21	22								
8.5	1.000										
9.0	1.000										
9.5	0.999	1.000									
10.0	0.998	0.999	1.000								

Table 2 Poisson Distribution Function (Continued)

$\lambda \backslash x$	0	1	2	3	4	5	6	7	8	9
10.5	0.000	0.000	0.002	0.007	0.021	0.050	0.102	0.179	0.279	0.397
11.0	0.000	0.000	0.001	0.005	0.015	0.038	0.079	0.143	0.232	0.341
11.5	0.000	0.000	0.001	0.003	0.011	0.028	0.060	0.114	0.191	0.289
12.0	0.000	0.000	0.001	0.002	0.008	0.020	0.046	0.090	0.155	0.242
12.5	0.000	0.000	0.000	0.002	0.005	0.015	0.035	0.070	0.125	0.201
13.0	0.000	0.000	0.000	0.001	0.004	0.011	0.026	0.054	0.100	0.166
13.5	0.000	0.000	0.000	0.001	0.003	0.008	0.019	0.041	0.079	0.135
14.0	0.000	0.000	0.000	0.000	0.002	0.006	0.014	0.032	0.062	0.109
14.5	0.000	0.000	0.000	0.000	0.001	0.004	0.010	0.024	0.048	0.088
15.0	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.018	0.037	0.070
	10	11	12	13	14	15	16	17	18	19
10.5	0.521	0.639	0.742	0.825	0.888	0.932	0.960	0.978	0.988	0.994
11.0	0.460	0.579	0.689	0.781	0.854	0.907	0.944	0.968	0.982	0.991
11.5	0.402	0.520	0.633	0.733	0.815	0.878	0.924	0.954	0.974	0.986
12.0	0.347	0.462	0.576	0.682	0.772	0.844	0.899	0.937	0.963	0.979
12.5	0.297	0.406	0.519	0.628	0.725	0.806	0.869	0.916	0.948	0.969
13.0	0.252	0.353	0.463	0.573	0.675	0.764	0.835	0.890	0.930	0.957
13.5	0.211	0.304	0.409	0.518	0.623	0.718	0.798	0.861	0.908	0.942
14.0	0.176	0.260	0.358	0.464	0.570	0.669	0.756	0.827	0.883	0.923
14.5	0.145	0.220	0.311	0.413	0.518	0.619	0.711	0.790	0.853	0.901
15.0	0.118	0.185	0.268	0.363	0.466	0.568	0.664	0.749	0.819	0.875
	20	21	22	23	24	25	26	27	28	29
10.5	0.997	0.999	0.999	1.000						
11.0	0.995	0.998	0.999	1.000						
11.5	0.992	0.996	0.998	0.999	1.000					
12.0	0.988	0.994	0.997	0.999	0.999	1.000				
12.5	0.983	0.991	0.995	0.998	0.999	0.999	1.000			
13.0	0.975	0.986	0.992	0.996	0.998	0.999	1.000			
13.5	0.965	0.980	0.989	0.994	0.997	0.998	0.999	1.000		
14.0	0.952	0.971	0.983	0.991	0.995	0.997	0.999	0.999	1.000	
14.5	0.936	0.960	0.976	0.986	0.992	0.996	0.998	0.999	0.999	1.000
15.0	0.917	0.947	0.967	0.981	0.989	0.994	0.997	0.998	0.999	1.000

Table 2
Poisson Distribution Function (Continued)

$\lambda \backslash x$	4	5	6	7	8	9	10	11	12	13
16	0.000	0.001	0.004	0.010	0.022	0.043	0.077	0.127	0.193	0.275
17	0.000	0.001	0.002	0.005	0.013	0.026	0.049	0.085	0.135	0.201
18	0.000	0.000	0.001	0.003	0.007	0.015	0.030	0.055	0.092	0.143
19	0.000	0.000	0.001	0.002	0.004	0.009	0.018	0.035	0.061	0.098
20	0.000	0.000	0.000	0.001	0.002	0.005	0.011	0.021	0.039	0.066
21	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.013	0.025	0.043
22	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.008	0.015	0.028
23	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.009	0.017
24	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.005	0.011
25	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.006
	14	15	16	17	18	19	20	21	22	23
16	0.368	0.467	0.566	0.659	0.742	0.812	0.868	0.911	0.942	0.963
17	0.281	0.371	0.468	0.564	0.655	0.736	0.805	0.861	0.905	0.937
18	0.208	0.287	0.375	0.469	0.562	0.651	0.731	0.799	0.855	0.899
19	0.150	0.215	0.292	0.378	0.469	0.561	0.647	0.725	0.793	0.849
20	0.105	0.157	0.221	0.297	0.381	0.470	0.559	0.644	0.721	0.787
21	0.072	0.111	0.163	0.227	0.302	0.384	0.471	0.558	0.640	0.716
22	0.048	0.077	0.117	0.169	0.232	0.306	0.387	0.472	0.556	0.637
23	0.031	0.052	0.082	0.123	0.175	0.238	0.310	0.389	0.472	0.555
24	0.020	0.034	0.056	0.087	0.128	0.180	0.243	0.314	0.392	0.473
25	0.012	0.022	0.038	0.060	0.092	0.134	0.185	0.247	0.318	0.394
	24	25	26	27	28	29	30	31	32	33
16	0.978	0.987	0.993	0.996	0.998	0.999	0.999	1.000		
17	0.959	0.975	0.985	0.991	0.995	0.997	0.999	0.999	1.000	
18	0.932	0.955	0.972	0.983	0.990	0.994	0.997	0.998	0.999	1.000
19	0.893	0.927	0.951	0.969	0.980	0.988	0.993	0.996	0.998	0.999
20	0.843	0.888	0.922	0.948	0.966	0.978	0.987	0.992	0.995	0.997
21	0.782	0.838	0.883	0.917	0.944	0.963	0.976	0.985	0.991	0.994
22	0.712	0.777	0.832	0.877	0.913	0.940	0.959	0.973	0.983	0.989
23	0.635	0.708	0.772	0.827	0.873	0.908	0.936	0.956	0.971	0.981
24	0.554	0.632	0.704	0.768	0.823	0.868	0.904	0.932	0.953	0.969
25	0.473	0.553	0.629	0.700	0.763	0.818	0.863	0.900	0.929	0.950
	34	35	36	37	38	39	40	41	42	43
19	0.999	1.000								
20	0.999	0.999	1.000							
21	0.997	0.998	0.999	0.999	1.000					
22	0.994	0.996	0.998	0.999	0.999	1.000				
23	0.998	0.993	0.996	0.997	0.999	0.999	1.000			
24	0.979	0.987	0.992	0.995	0.997	0.998	0.999	0.999	1.000	
25	0.966	0.978	0.985	0.991	0.994	0.997	0.998	0.999	1.000	