STATISTICAL PROCESS CONTROL

Control Charts

Yield information leading to decisions, such as

- **Make-or-buy**
- **Plant improvements**
- **Process improvements**
- **Contractual agreements with suppliers**

Quality-related methods and philosophies include:

Reliability analysis

Maintainability analysis

Design of Experiments

Acceptance sampling models

Total Quality Management

Pareto diagrams

Fishbone (cause/effect) diagrams

Taguchi methods

Deming's 14 Points

Control charting

General statistical methods

Process centering

Control Charts

- ***Chance*** **causes**

  = inherent variability

  ⇒ stable system

- ***Assignable*** **causes**

  = process problems

  ⇒ out-of-control system

OBJECTIVE:

reduction of process variability resulting from assignable causes

Control Charts

***Essence* of control charting is:**

- **Establishing Confidence Intervals**
- **Selecting samples and determining their positions within those intervals**
- **Identifying patterns that suggest the process may be out of control**
- **Using the patterns to assist in determining assignable causes**

Many Types of Control Charts

(e.g., Variables, Fraction Defective, Defects per Unit)

~ Variables Control Charts ~

- **rational (measurement) data**
- **means, ranges, variances**
- **Gaussian (i.e., Normal) distribution assumptions**
- **parameters (e.g., μ, σ) can be estimated**
- **3σ control limits (i.e., 99% confidence interval)**

We consider only the variables control charts for means and ranges

Variables Control Charts

Tabled Values for 3σ Control Limits

	Factor for Control Limits						
	$\bar{X}$ Chart			R Chart		S Chart	
n*	A_1	A_2	d_2	D_3	D_4	c_4	n
2	3.760	1.880	1.128	0	3.267	0.7979	2
3	2.394	1.023	1.693	0	2.575	0.8862	3
4	1.880	.729	2.059	0	2.282	0.9213	4
5	1.596	.577	2.326	0	2.115	0.9400	5
6	1.410	.483	2.534	0	2.004	0.9515	6
7	1.277	.419	2.704	.076	1.924	0.9594	7
8	1.175	.373	2.847	.136	1.864	0.9650	8
9	1.094	.337	2.970	.184	1.816	0.9693	9
10	1.028	.308	3.078	.223	1.777	0.9727	10
11	.973	.285	3.173	.256	1.744	0.9754	11
12	.925	.266	3.258	.284	1.716	0.9776	12
13	.884	.249	3.336	.308	1.692	0.9794	13
14	.848	.235	3.407	.329	1.671	0.9810	14
15	.816	.223	3.472	.348	1.652	0.9823	15
16	.788	.212	3.532	.364	1.636	0.9835	16
17	.762	.203	3.588	.379	1.621	0.9845	17
18	.738	.194	3.640	.392	1.608	0.9854	18
19	.717	.187	3.689	.404	1.596	0.9862	19
20	.697	.180	3.735	.414	1.586	0.9869	20
21	.679	.173	3.778	.425	1.575	0.9876	21
22	.662	.167	3.819	.434	1.566	0.9882	22
23	.647	.162	3.858	.443	1.557	0.9887	23
24	.632	.157	3.895	.452	1.548	0.9892	24
25	.619	.153	3.931	.459	1.541	0.9896	25

*$n > 25$: $A_1 = 3/\sqrt{n}$ where n = number of observations in sample.

Control Chart for Ranges

$\bar{R}$ Chart

for process variability control

1. Select m samples, each of size n
2. Determine the range, R_i, of each of m samples
3. Determine average range,
$$\bar{R} = \frac{1}{m}\sum_{i=1}^{m} R_i$$
4. Determine D_3 and D_4 from tabled values
5. Now
$$UCL = D_4\bar{R} \quad \text{upper control limit}$$
$$CL = \bar{R} \quad \text{center line}$$
$$LCL = D_3\bar{R} \quad \text{lower control limit}$$
6. Delete assignable outliers and recompute limits

$\bar{R}$ Chart - Example

Subgroup Number	Date	Time	Measurements X_1	X_2	X_3	X_4	Average $\bar{X}$	Range R	Comments
1	12/23	8:50	35	40	32	33	6.35	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary operator
5		4:20	38	34	44	40	6.39	0.10	
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	52	49	51	6.50	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	33	32	37	38	6.35	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

Data Set -

m = # samples (subgroups) = 25

n = # items / subgroup = 4

$\Sigma R_i = 2.19$

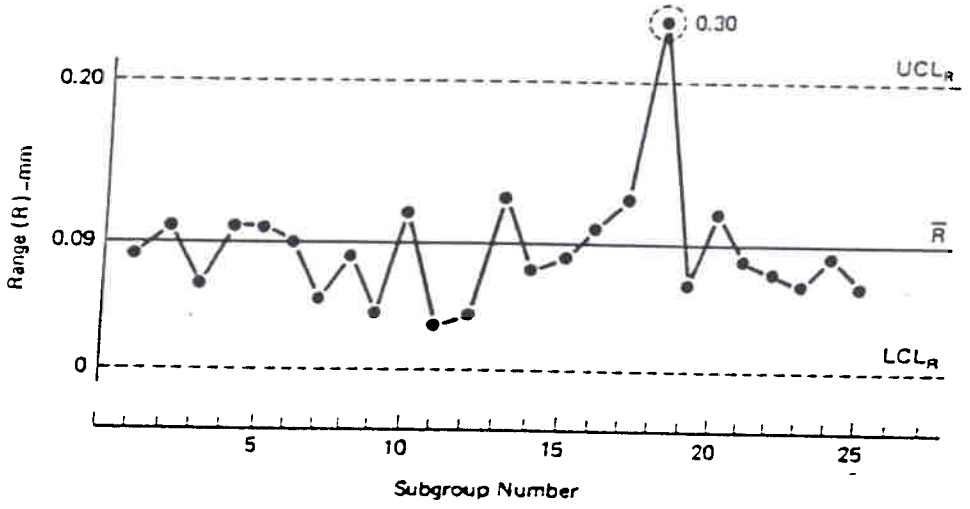
Note assignable cause comments for subgroups 4, 18 and 20

$\bar{R}$ Chart - Example

$n = 4 \qquad m = 25 \qquad \Sigma R_i = 2.19$

average range, $\bar{R} = 2.19/25 = 0.09$
from tabled values for $n = 4$,
$D_3 = 0 \qquad D_4 = 2.282$

$UCL = D_4 \bar{R} = (2.282)(0.09) = 0.20$
$CL = \bar{R} = 0.09$
$LCL = D_3 \bar{R} = (0)(0.09) = 0$



Note outlier: $R_{18} = 0.30$, damaged oil line

Delete assignable outlier R_{18}, so $\bar{R} = 0.079$

$UCL = D_4 \bar{R} = (2.282)(0.079) = 0.18$
$CL = \bar{R} = 0.079$
$LCL = D_3 \bar{R} = (0)(0.079) = 0$

Control Chart for Means

$\bar{X}$ Chart

for average quality control

1. Select m samples, each of size n
2. Determine the mean, $\bar{x}_i = \frac{1}{n}\sum_{i=1}^{n} x_i$, of each sample
3. Determine the grand mean, $\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$, to estimate μ
4. Determine range, R_i, of each of m samples
5. Determine average range, $\bar{R} = \frac{1}{m}\sum_{i=1}^{m} R_i$
6. Determine A_2 $(= 3/d_2\sqrt{n})$; A_2 and d_2 in Table(5)
7. Now:
$$UCL = \bar{\bar{x}} + A_2\bar{R}$$
$$CL = \bar{\bar{x}}$$
$$LCL = \bar{\bar{x}} - A_2\bar{R}$$
8. If outliers are assignable, delete values and recompute limits

$\bar{X}$ Chart - Example

Subgroup Number	Date	Time	Measurements X_1	X_2	X_3	X_4	Average $\bar{X}$	Range R	Comments
1	12/23	8:50	35	40	32	33	6.35	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary operator
5		4:20	38	34	44	40	6.39	0.10	
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	52	49	51	6.50	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	33	32	37	38	6.35	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
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23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

$m = 25 \qquad \Sigma \bar{x}_i = 160.25 \qquad \Sigma R_i = 2.19$

grand mean, $\bar{\bar{x}} = 160.25/25 = 6.41$

average range, $\bar{R} = 2.19/25 = 0.09$

subgroup size $= n = 4 \Rightarrow A_2 = 0.729$ from table

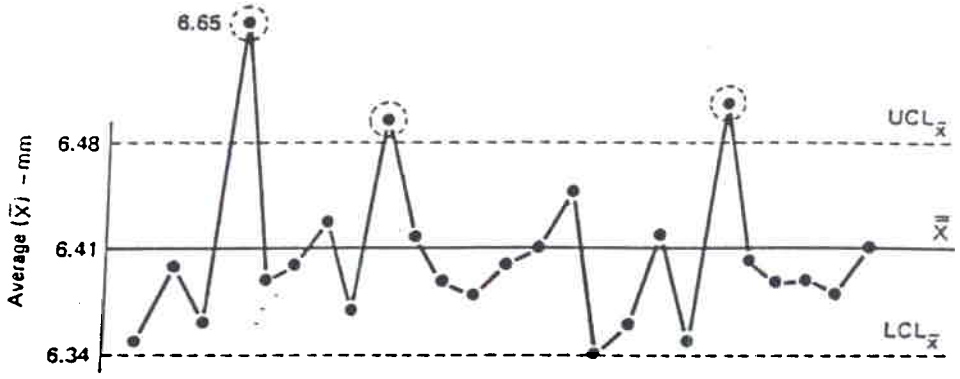
$UCL = \bar{\bar{x}} + A_2 \bar{R} = 6.41 + (0.729)(0.09) = 6.48$

$CL = \bar{\bar{x}} = 6.41$

$LCL = \bar{\bar{x}} - A_2 \bar{R} = 6.41 - (0.729)(0.09) = 6.34$

(CONTINUED)

$\bar{X}$ Chart - Example
(CONTINUED)



Note outliers: $\bar{x}_4 = 6.65$, new temp operator
$\bar{x}_9 = 6.50$, unassignable
$\bar{x}_{20} = 6.51$, bad material

Delete assignable outliers $\bar{x}_4$ and $\bar{x}_{20}$, and recompute $\bar{\bar{x}} = 6.40$

Using revised $\bar{R} = 0.079$ and revised $\bar{\bar{x}} = 6.40$:

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 6.40 + (0.729)(0.079) = 6.46$$
$$CL = \bar{\bar{x}} = 6.40$$
$$LCL = \bar{\bar{x}} - A_2\bar{R} = 6.40 - (0.729)(0.079) = 6.34$$

Note: ***There are two approaches used in establishing revised limits for Mean and Range Control Charts***

The approaches differ in the manner in which assignable cause data are addressed

Approach One:

Subgroups with assignable out-of-limit mean *and/or* range values are completely eliminated before determining revised grand mean *and* average range values.

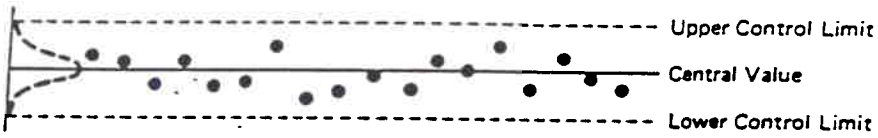
Approach Two:

Using the original set of subgroups, assignable out-of-limit subgroup range values are eliminated and then the revised average range value is determined.

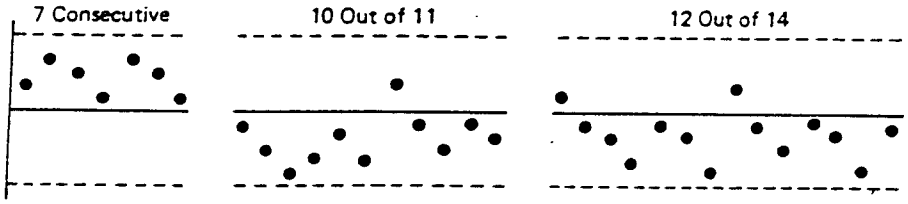
Again using the original set of subgroups, assignable out-of-limit subgroup mean values are eliminated and then the revised grand mean value is determined.

Control Charts

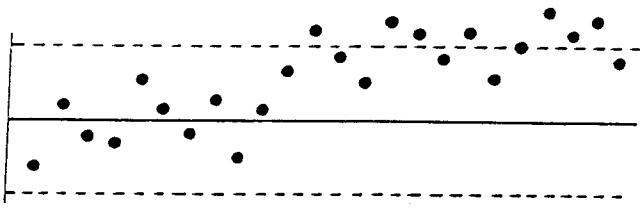
PATTERN VARIATIONS



Normal Pattern Variation



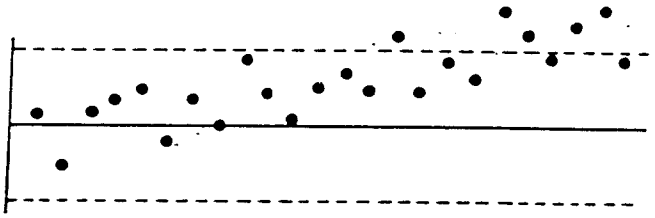
Out-of-Control: Runs Above or Below Mean



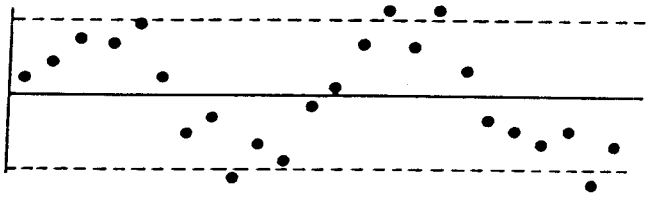
Out-of-Control: Change or Jump in Level

Control Charts

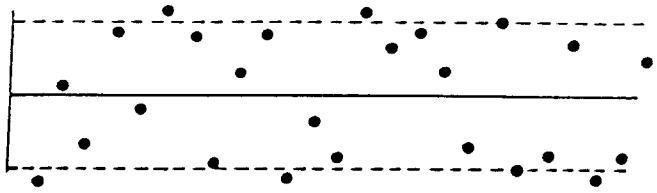
PATTERN VARIATIONS



Out-of-Control: Trend or Steady State Change



Out-of-Control: Recurring Cycles



Out-of-Control: Two Universes