PROBABILITY DENSITIES

CONTINUOUS DISTRIBUTIONS

NORMAL DISTRIBUTION

NORMAL (GAUSSIAN) DISTRIBUTION

Introduced by C. F. Gauss in connection with the theory of errors of physical measurements

The most important continuous distribution, for the following reasons:

- **Many random variables that appear in connection with practical experiments or observations are normally distributed, and many other types of variables are approximately normally distributed**

- **Even if a variable is neither normally nor approximately normally distributed, it can frequently be transformed into a normally distributed variable by a relatively simple transformation**

- **Certain more complicated distributions can be approximated by the normal distribution (e.g., the binomial distribution).**

- **Several variables that are used in statistical inference are normally or approximately normally distributed.**

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE APPLICATIONS

- **Given that the height of a population is a normal random variable with mean 68 inches and standard deviation 2.5 inches. If we select 200 of these persons at random, how many of them can we expect to be (a) 68 inches, (b) greater than 70 inches, assuming that the measurements are recorded to the nearest inch?**

- **Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 milliamperes squared. What is the probability that a measurement will exceed 1.3 milliamperes?**

- **The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce. Determine the specifications that are symmetric about the mean that include 99% of all cans.**

- **The sick-leave time of employees in a firm in a month is normally distributed with a mean of 100 hours and a standard deviation of 20 hours. How much time should be budgeted for sick leave if the budgeted amount should be exceeded with a probability of only 10%?**

NORMAL (GAUSSIAN) DISTRIBUTION

also written as $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

Probability Density Function:

$$f(x;\mu,\sigma) = (\sigma\sqrt{2\pi})^{-1} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

$$-\infty < x < \infty \; ; \; -\infty < \mu < \infty \; ; \; \sigma > 0$$

Distribution Function:

$$F(x;\mu,\sigma) = (\sigma\sqrt{2\pi})^{-1} \int_{-\infty}^{x} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\} dy$$

$P(X \leq x)$

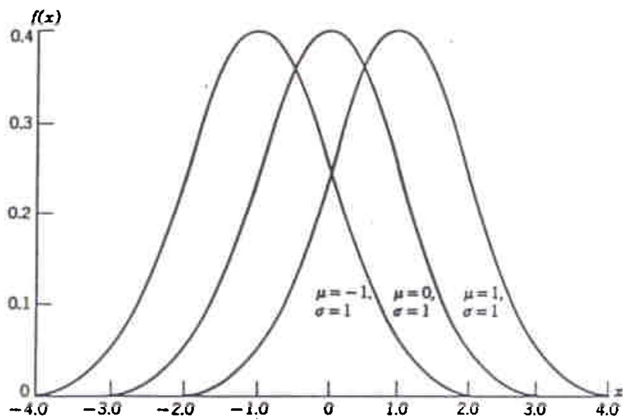
Mean and Variance:

mean $= \mu$

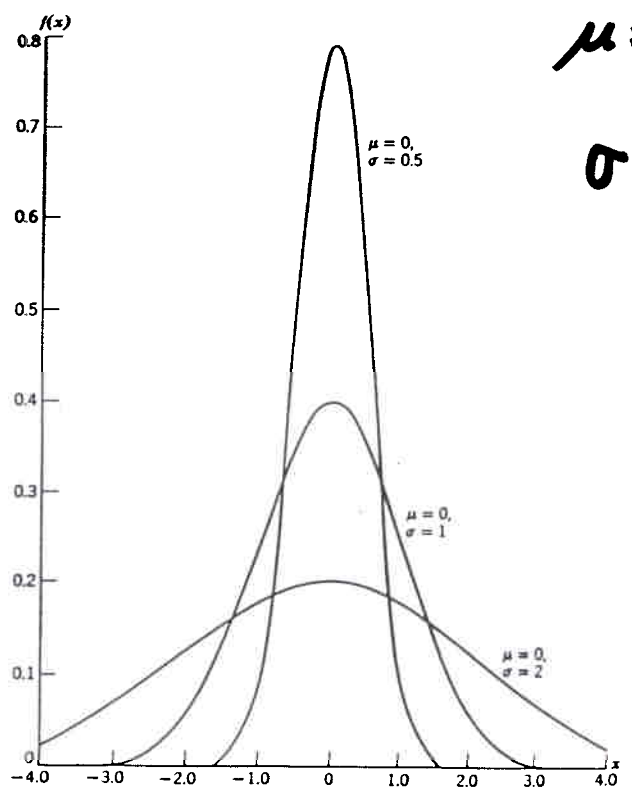
variance $= \sigma^2$

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBABILITY DENSITY FUNCTIONS



Normal distributions with different values of μ and same σ.



Normal distributions with different values of σ and same μ.

μ = location parameter

σ = ~~scale~~ parameter

NORMAL (GAUSSIAN) DISTRIBUTION

DETERMINING PROBABILITIES:

$$\Pr(a < X \leq b)$$

$$= F(b) - F(a)$$

$$= (\sigma\sqrt{2\pi})^{-1} \int_a^b \exp -\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2 dy$$

STANDARD NORMAL VARIATE:

If x is normally distributed with arbitrary mean and variance, then the standardized or standard normal variate

$$z = (x - \mu) / \sigma$$

is normally distributed with $\mu = 0$ and $\sigma = 1$. This allows us to use a single table to evaluate cumulative probabilities for any normal distribution.

z = # Standard deviations from the mean

NORMAL (GAUSSIAN) DISTRIBUTION

TABLED VALUES: (Source: Montgomery + Runger)

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

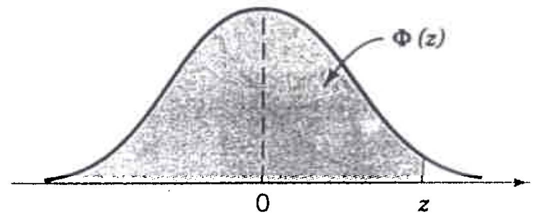


Table II Cumulative Standard Normal Distribution (*continued*)

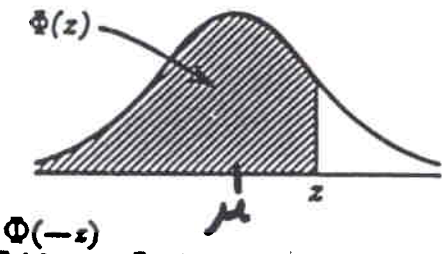
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.52790
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.5674
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.6
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.71226	
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.74537	
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.77637	
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.8961	
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.91	
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.	
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939		
1.6	0.945201	0.946301	0.9473					
1.7	0.955435	0.956367	0.					
1.8	0.964070	0.964852	0.9					
1.9	0.971283	0.971933	0.972					
2.0	0.977250	0.977784	0.					
2.1								

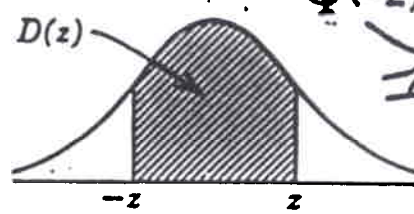
ab/prob 8-6a

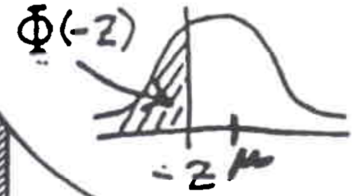
3 Normal Distribution

Source: Kreyszig

Table 3a. Distribution Function (3), Sec. 8.2







$D(z) = \Phi(z) - \Phi(-z)$

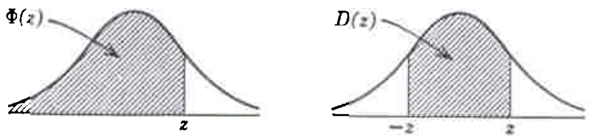
$\Phi(-z) = 1 - \Phi(z), \qquad \Phi(0) = 0.5$

More extended tables: National Bureau of Standards (1953), Hald (1962). Index for further tables: Greenwood and Hartley (1961) (cf. Appendix 3).

z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$
	0.	0.	0.		0.	0.	0.		0.	0.	0.
0.01	4960	5040	0080	0.51	3050	6950	3899	1.01	1562	8438	6875
0.02	4920	5080	0160	0.52	3015	6985	3969	1.02	1539	8461	6923
0.03	4880	5120	0239	0.53	2981	7019	4039	1.03	1515	8485	6970
0.04	4840	5160	0319	0.54	2946	7054	4108	1.04	1492	8508	7017
0.05	4801	5199	0399	0.55	2912	7088	4177	1.05	1469	8531	7063
0.06	4761	5239	0478	0.56	2877	7123	4245	1.06	1446	8554	7109
0.07	4721	5279	0558	0.57	2843	7157	4313	1.07	1423	8577	7154
0.08	4681	5319	0638	0.58	2810	7190	4381	1.08	1401	8599	7199
0.09	4641	5359	0717	0.59	2776	7224	4448	1.09	1379	8621	7243
0.10	4602	5398	0797	0.60	2743	7257	4515	1.10	1357	8643	7287
0.11	4562	5438	0876	0.61	2709	7291	4581	1.11	1335	8665	7330
0.12	4522	5478	0955	0.62	2676	7324	4647	1.12	1314	8686	7373
0.13	4483	5517	1034	0.63	2643	7357	4713	1.13	1292	8708	7415
0.14	4443	5557	1113	0.64	2611	7389	4778	1.14	1271	8729	7457
0.15	4404	5596	1192	0.65	2578	7422	4843	1.15	1251	8749	7499
0.16	4364	5636	1271	0.66	2546	7454	4907	1.16	1230	8770	7540
[illegible]	[illegible]	5675	1350	0.67	2514	7486	4971	1.17	1210	8790	7580
			1428	0.68	2483	7517	5035	1.18	1190	8810	7620
			1507	0.69	2451	7549	5098	1.19	1170	8830	7660
			1585	0.70	2420	7580	5161	1.20	1151	8849	7699
				0.71	2389	7611	5223	1.21	1131	8869	7737
					2358	7642	5285	1.22	1112	8888	[illegible]
					[illegible]	7673	5346	1.23	1093	8907	
					[illegible]		5407	1.24			

3 Normal Distribution

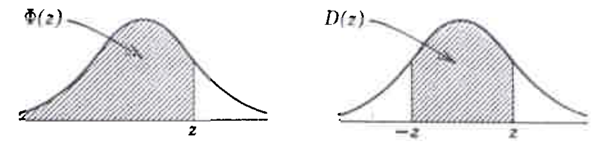
Table 3a. Distribution Function (3), Sec. 8.2



$D(z) = \Phi(z) - \Phi(-z)$
$\Phi(-z) = 1 - \Phi(z)$, $\Phi(0) = 0.5$
More extended tables: National Bureau of Standards (1953), Hald (1962). Index for further tables: Greenwood and Hartley (1961) (cf. Appendix 3).

z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$
	0.	0.	0.		0.	0.	0.		0.	0.	0.
0.01	4960	5040	0080	0.51	3050	6950	3899	1.01	1562	8438	6875
0.02	4920	5080	0160	0.52	3015	6985	3969	1.02	1539	8461	6923
0.03	4880	5120	0239	0.53	2981	7019	4039	1.03	1515	8485	6970
0.04	4840	5160	0319	0.54	2946	7054	4108	1.04	1492	8508	7017
0.05	4801	5199	0399	0.55	2912	7088	4177	1.05	1469	8531	7063
0.06	4761	5239	0478	0.56	2877	7123	4245	1.06	1446	8554	7109
0.07	4721	5279	0558	0.57	2843	7157	4313	1.07	1423	8577	7154
0.08	4681	5319	0638	0.58	2810	7190	4381	1.08	1401	8599	7199
0.09	4641	5359	0717	0.59	2776	7224	4448	1.09	1379	8621	7243
0.10	4602	5398	0797	0.60	2743	7257	4515	1.10	1357	8643	7287
0.11	4562	5438	0876	0.61	2709	7291	4581	1.11	1335	8665	7330
0.12	4522	5478	0955	0.62	2676	7324	4647	1.12	1314	8686	7373
0.13	4483	5517	1034	0.63	2643	7357	4713	1.13	1292	8708	7415
0.14	4443	5557	1113	0.64	2611	7389	4778	1.14	1271	8729	7457
0.15	4404	5596	1192	0.65	2578	7422	4843	1.15	1251	8749	7499
0.16	4364	5636	1271	0.66	2546	7454	4907	1.16	1230	8770	7540
0.17	4325	5675	1350	0.67	2514	7486	4971	1.17	1210	8790	7580
0.18	4286	5714	1428	0.68	2483	7517	5035	1.18	1190	8810	7620
0.19	4247	5753	1507	0.69	2451	7549	5098	1.19	1170	8830	7660
0.20	4207	5793	1585	0.70	2420	7580	5161	1.20	1151	8849	7699
0.21	4168	5832	1663	0.71	2389	7611	5223	1.21	1131	8869	7737
0.22	4129	5871	1741	0.72	2358	7642	5285	1.22	1112	8888	7775
0.23	4090	5910	1819	0.73	2327	7673	5346	1.23	1093	8907	7813
0.24	4052	5948	1897	0.74	2296	7704	5407	1.24	1075	8925	7850
0.25	4013	5987	1974	0.75	2266	7734	5467	1.25	1056	8944	7887
0.26	3974	6026	2051	0.76	2236	7764	5527	1.26	1038	8962	7923
0.27	3936	6064	2128	0.77	2206	7794	5587	1.27	1020	8980	7959
0.28	3897	6103	2205	0.78	2177	7823	5646	1.28	1003	8997	7995
0.29	3859	6141	2282	0.79	2148	7852	5705	1.29	0985	9015	8029
0.30	3821	6179	2358	0.80	2119	7881	5763	1.30	0968	9032	8064
0.31	3783	6217	2434	0.81	2090	7910	5821	1.31	0951	9049	8098
0.32	3745	6255	2510	0.82	2061	7939	5878	1.32	0934	9066	8132
0.33	3707	6293	2586	0.83	2033	7967	5935	1.33	0918	9082	8165
0.34	3669	6331	2661	0.84	2005	7995	5991	1.34	0901	9099	8198
0.35	3632	6368	2737	0.85	1977	8023	6047	1.35	0885	9115	8230
0.36	3594	6406	2812	0.86	1949	8051	6102	1.36	0869	9131	8262
0.37	3557	6443	2886	0.87	1922	8078	6157	1.37	0853	9147	8293
0.38	3520	6480	2961	0.88	1894	8106	6211	1.38	0838	9162	8324
0.39	3483	6517	3035	0.89	1867	8133	6265	1.39	0823	9177	8355
0.40	3446	6554	3108	0.90	1841	8159	6319	1.40	0808	9192	8385
0.41	3409	6591	3182	0.91	1814	8186	6372	1.41	0793	9207	8415
0.42	3372	6628	3255	0.92	1788	8212	6424	1.42	0778	9222	8444
0.43	3336	6664	3328	0.93	1762	8238	6476	1.43	0764	9236	8473
0.44	3300	6700	3401	0.94	1736	8264	6528	1.44	0749	9251	8501
0.45	3264	6736	3473	0.95	1711	8289	6579	1.45	0735	9265	8529
0.46	3228	6772	3545	0.96	1685	8315	6629	1.46	0721	9279	8557
0.47	3192	6808	3616	0.97	1660	8340	6680	1.47	0708	9292	8584
0.48	3156	6844	3688	0.98	1635	8365	6729	1.48	0694	9306	8611
0.49	3121	6879	3759	0.99	1611	8389	6778	1.49	0681	9319	8638
0.50	3085	6915	3829	1.00	1587	8413	6827	1.50	0668	9332	8664

Table 3a. Distribution Function (3), Sec. 8.2 (*Continued*)



z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$	z	$\Phi(-z)$	$\Phi(z)$	$D(z)$
	0.	0.	0.		0.	0.	0.		0.	0.	0.
1.51	0655	9345	8690	2.01	0222	9778	9556	2.51	0060	9940	9879
1.52	0643	9357	8715	2.02	0217	9783	9566	2.52	0059	9941	9883
1.53	0630	9370	8740	2.03	0212	9788	9576	2.53	0057	9943	9886
1.54	0618	9382	8764	2.04	0207	9793	9586	2.54	0055	9945	9889
1.55	0606	9394	8789	2.05	0202	9798	9596	2.55	0054	9946	9892
1.56	0594	9406	8812	2.06	0197	9803	9606	2.56	0052	9948	9895
1.57	0582	9418	8836	2.07	0192	9808	9615	2.57	0051	9949	9898
1.58	0571	9429	8859	2.08	0188	9812	9625	2.58	0049	9951	9901
1.59	0559	9441	8882	2.09	0183	9817	9634	2.59	0048	9952	9904
1.60	0548	9452	8904	2.10	0179	9821	9643	2.60	0047	9953	9907
1.61	0537	9463	8926	2.11	0174	9826	9651	2.61	0045	9955	9909
1.62	0526	9474	8948	2.12	0170	9830	9660	2.62	0044	9956	9912
1.63	0516	9484	8969	2.13	0166	9834	9668	2.63	0043	9957	9915
1.64	0505	9495	8990	2.14	0162	9838	9676	2.64	0041	9959	9917
1.65	0495	9505	9011	2.15	0158	9842	9684	2.65	0040	9960	9920
1.66	0485	9515	9031	2.16	0154	9846	9692	2.66	0039	9961	9922
1.67	0475	9525	9051	2.17	0150	9850	9700	2.67	0038	9962	9924
1.68	0465	9535	9070	2.18	0146	9854	9707	2.68	0037	9963	9926
1.69	0455	9545	9090	2.19	0143	9857	9715	2.69	0036	9964	9929
1.70	0446	9554	9109	2.20	0139	9861	9722	2.70	0035	9965	9931
1.71	0436	9564	9127	2.21	0136	9864	9729	2.71	0034	9966	9933
1.72	0427	9573	9146	2.22	0132	9868	9736	2.72	0033	9967	9935
1.73	0418	9582	9164	2.23	0129	9871	9743	2.73	0032	9968	9937
1.74	0409	9591	9181	2.24	0125	9875	9749	2.74	0031	9969	9939
1.75	0401	9599	9199	2.25	0122	9878	9756	2.75	0030	9970	9940
1.76	0392	9608	9216	2.26	0119	9881	9762	2.76	0029	9971	9942
1.77	0384	9616	9233	2.27	0116	9884	9768	2.77	0028	9972	9944
1.78	0375	9625	9249	2.28	0113	9887	9774	2.78	0027	9973	9946
1.79	0367	9633	9265	2.29	0110	9890	9780	2.79	0026	9974	9947
1.80	0359	9641	9281	2.30	0107	9893	9786	2.80	0026	9974	9949
1.81	0351	9649	9297	2.31	0104	9896	9791	2.81	0025	9975	9950
1.82	0344	9656	9312	2.32	0102	9898	9797	2.82	0024	9976	9952
1.83	0336	9664	9328	2.33	0099	9901	9802	2.83	0023	9977	9953
1.84	0329	9671	9342	2.34	0096	9904	9807	2.84	0023	9977	9955
1.85	0322	9678	9357	2.35	0094	9906	9812	2.85	0022	9978	9956
1.86	0314	9686	9371	2.36	0091	9909	9817	2.86	0021	9979	9958
1.87	0307	9693	9385	2.37	0089	9911	9822	2.87	0021	9979	9959
1.88	0301	9699	9399	2.38	0087	9913	9827	2.88	0020	9980	9960
1.89	0294	9706	9412	2.39	0084	9916	9832	2.89	0019	9981	9961
1.90	0287	9713	9426	2.40	0082	9918	9836	2.90	0019	9981	9963
1.91	0281	9719	9439	2.41	0080	9920	9840	2.91	0018	9982	9964
1.92	0274	9726	9451	2.42	0078	9922	9845	2.92	0018	9982	9965
1.93	0268	9732	9464	2.43	0075	9925	9849	2.93	0017	9983	9966
1.94	0262	9738	9476	2.44	0073	9927	9853	2.94	0016	9984	9967
1.95	0256	9744	9488	2.45	0071	9929	9857	2.95	0016	9984	9968
1.96	0250	9750	9500	2.46	0069	9931	9861	2.96	0015	9985	9969
1.97	0244	9756	9512	2.47	0068	9932	9865	2.97	0015	9985	9970
1.98	0239	9761	9523	2.48	0066	9934	9869	2.98	0014	9986	9971
1.99	0233	9767	9534	2.49	0064	9936	9872	2.99	0014	9986	9972
2.00	0228	9772	9545	2.50	0062	9938	9876	3.00	0013	9987	9973

9

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBLEMS - - - -

If a random variable has the standard normal distribution, find the probability that it will take on a value

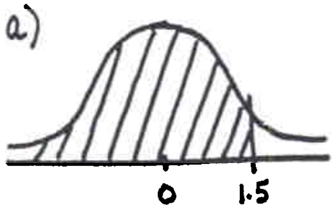
a) less than 1.50
b) less than -1.20
c) greater than 2.16
d) greater than -1.75

"z"

$Pr(X \le 1.50)$

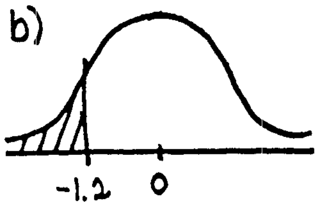
$\cong Pr(X < 1.50)$

a)



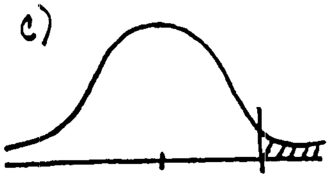
$Pr(X < 1.50) = 0.9332$ or 93.32%

b)



$Pr(X < -1.20) = 0.1151$

c)

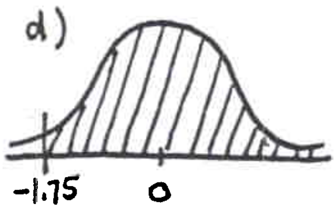


$Pr(X > 2.16) = 0.0154$

(1) $1 - Pr(X < 2.16)$

(2) $Pr(X \le -2.16)$

d)



$Pr(X > -1.75) = 0.9599$

(1) $1 - Pr(X \le -1.75)$

(2) $Pr(X \le 1.75)$

a61prob3-7b

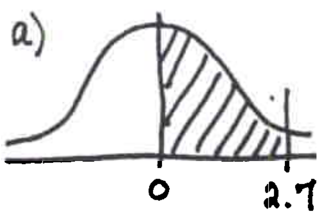
NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBLEMS - - - -

If a random variable has the standard normal distribution, find the probability that it will take on a value

a) between 0 and 2.7
b) between 1.22 and 2.43
c) between -1.35 and -0.35
d) between -1.70 and 1.35

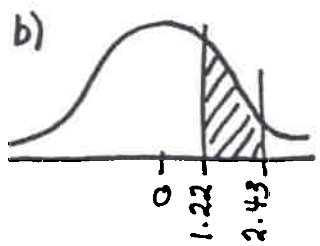
a)



$Pr(0 < X < 2.7) = 0.4965$

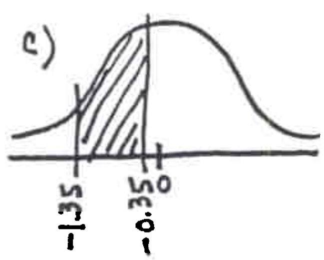
$Pr(X < 2.7) - Pr(X < 0)$

b)



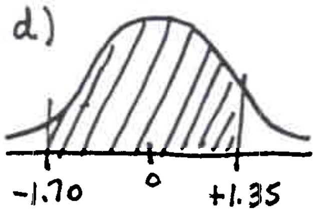
$Pr(1.22 < X < 2.43) = 0.1037$

c)



$Pr(-1.35 < X < -0.35) = 0.2747$

d)



$Pr(-1.70 < X < 1.35) = 0.8669$

a61prob3-8a

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBLEMS - - - -

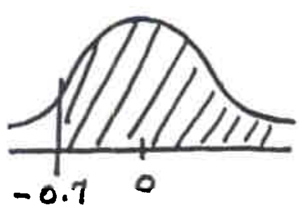
The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with $\mu = 12.9$ minutes and $\sigma = 2.0$ minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take

a) at least 11.5 minutes?
b) anywhere from 11.0 to 14.8 minutes?

Let X = assembly time ; $\mathcal{N} \sim (12.9, 4.0)$ normally distributed w/ mean = 12.9 and variance = 4.0

a) $Pr(X = \text{at least } 11.5 \text{ minutes})$
$= Pr(X \geq 11.5)$

$$z = \frac{x - \mu}{\sigma} = \frac{11.5 - 12.9}{2.0} = -0.7$$



$Pr(X \geq 11.5) = 0.758$
$Pr(z \geq -0.7) = 0.758$

b) $Pr(11.0 \leq X \leq 14.8)$
$z_l = (11.0 - 12.9)/2.0 = -0.95$
$z_r = (14.8 - 12.9)/2.0 = +0.95$

$Pr(11.0 \leq X \leq 14.8) = 0.658$

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBLEMS - - - -

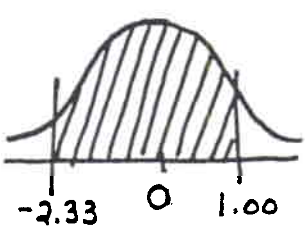
Specifications for a certain job call for washers with an inside diameter of 0.300 ± 0.005 inch. If the inside diameters of the washers supplied by a given manufacturer may be looked upon as a random variable having the normal distribution with $\mu = 0.302$ inch and $\sigma = 0.003$ inch, what percentage of these washers will meet specifications?

Upper limit = 0.300 + 0.005 = 0.305

Lower limit = 0.300 − 0.005 = 0.295

$z_u = (0.305 - 0.302)/0.003 = 1.00$

$z_l = (0.295 - 0.302)/0.003 = -2.33$



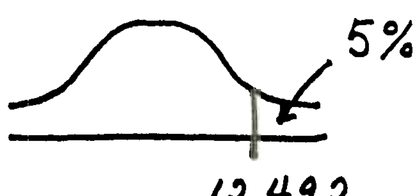
$Pr(0.295 \leq x \leq 0.305)$

$= 0.8314$

NORMAL (GAUSSIAN) DISTRIBUTION

EXAMPLE PROBLEMS: (Source: Zuwaylif)

A food processor packages instant coffee in small jars. The weights of the jars are normally distributed with a standard deviation of 0.3 ounce. If 5% of the jars weigh more than 12.492 ounces, what is the mean weight of the jars?



Let X denote jar weight

For the known x,

$$z = \frac{12.492 - \mu}{0.3}$$

$Pr\,(X > 12.492) = 0.05$

$\Rightarrow Pr\,(X \leq 12.492) = 1 - 0.05 = 0.95$

From the Table, 0.95 corresponds to

$z = 1.645$

Thus $1.645 = \dfrac{12.492 - \mu}{0.3}$

$\Rightarrow \mu \doteq 11.999$ or 12 ounces

NORMAL (GAUSSIAN) DISTRIBUTION

TRANSFORMATION OF DATA

Although observation values may not be Normally distributed, it is possible that some transformation of the data set might be Normally distributed

If the transformed data can be demonstrated to be approximately Normally distributed, then the transformed data set can be evaluated and inferences drawn

Typical transformations attempted include:

x^2 $\qquad$ $x^{1/2}$

x^3 $\qquad$ $-x^{-1}$

$x^{1/4}$ $\qquad$ $\ln x$

EXPONENTIAL DISTRIBUTION

- Simplest distribution describing failure times; assumes probability of failure at all ages is constant

- Used extensively with problems of reliability and maintainability

- Can be derived from the Poisson distribution by dealing with the average time between incidents (e.g., failures, accidents, or arrivals), instead of the average number of incidents per unit time

- Like the Poisson distribution, the exponential distribution has a "memoryless" property, since future behavior is independent of present or past behavior

- Although most often used in conjunction with problems involving the dimension of time, is also used with problems involving distance and occasionally other measurement dimensions

- **The lifetime of a particular electronic component is known to be exponentially distributed with a mean life of 1000 hours. What proportion of such components will fail before 800 hours?**

- **The determination of the length of the time interval between vehicle arrivals is important to the traffic engineer. If this length is too short, a vehicle attempting to merge or cross the traffic stream will be forced either to remain stationary or to interrupt the traffic stream. Wishing to predict the probability of a certain time between successive cars, a traffic engineer collected data of the gaps in traffic at a certain point on a freeway. From these data, she found that the mean gap length of time between successive cars is 8.33 seconds. What is the probability of a gap length of 9 to 11 seconds?**

Exponential Distribution -

Density function:

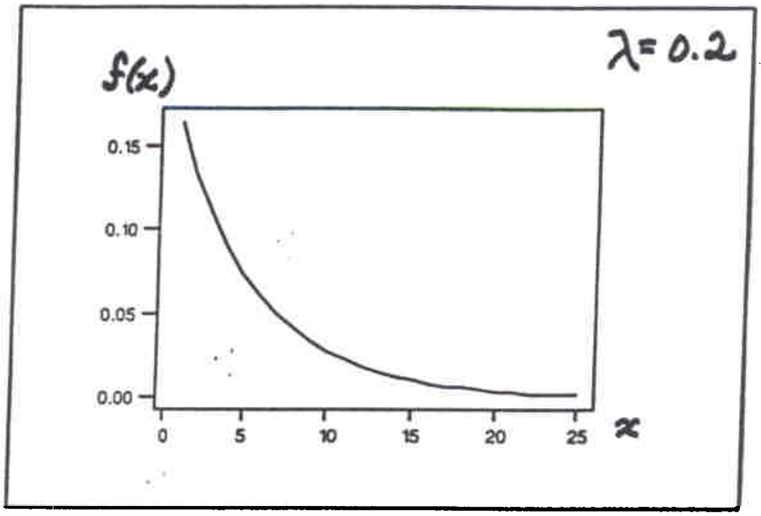
$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

Distribution function:

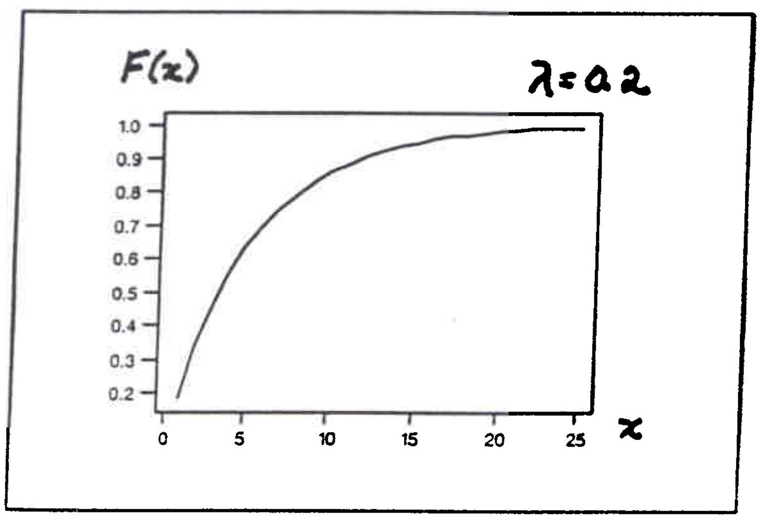
$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0$$

Mean: $\mu = 1/\lambda$

Variance: $\sigma^2 = 1/\lambda^2$



Exponential density function



Exponential distribution function

Exponential Distribution - Example

The life of a certain brand of light bulb can be approximated by the exponential probability density function. If the mean life of the light bulb is 1,000 hours, determine the probability that the bulb will last more than 1,000 hours.

Interpret as:

Determine the probability that a light bulb will have a life of 1000 or fewer hours, when μ = 1000, and take the complement.

$$\mu = 1000 = 1/\lambda \Rightarrow \lambda = 0.001$$

$$F(x \leq 1000) = \int_0^{1000} 0.001\, e^{-0.001x}\, dx$$

$$= 1 - e^{-(0.001)(1000)} = 1 - e^{-1}$$

$$F(x > 1000) = 1 - F(x \leq 1000)$$

$$= 1 - [1 - e^{-1}]$$

$$= e^{-1} = 0.368$$