Selected Reliability Concepts

Def$^{\underline{n}}$ of Reliability

"The probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered."

Four basic parts —
all require expert professional judgment

1. Probability
2. Adequate performance
3. Time intended
4. Operating conditions

Revisit
Basic Probability Concepts

GIVEN: Events A and B

Probability of-

- Occurrence of both Events when they are Independent

$$P(A \cap B) = P(A) * P(B)$$

- Occurrence of both Events when they are Dependent

$$P(A \cap B) = P(A|B) * P(B)$$
$$= P(B|A) * P(A)$$

- Occurrence of at least one of the two Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Occurrence of only one of the two Events

$$P(A \cup B) = P(A) + P(B)$$
$$\text{iff } P(A \cap B) = 0$$

Notation

= Reliability

= Probability of "Success"

= Probability of Failure

The conditions of "Success" and Failure are taken to be Mutually Exclusive

Thus

$$P(\text{Success}) + P(\text{Failure}) = 1$$

Or

$$R = 1 - Q$$

and

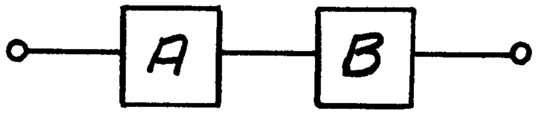
$$Q = 1 - R$$

Reliability Network Models

Non-Redundant Serial System

System Success
$\Rightarrow$ Success of all Components

System Failure
$\Rightarrow$ Failure of at least one Component


$\Rightarrow A \cap B$

Let $R_A = P(A \text{ succeeds})$
$\Rightarrow Q_A = P(A \text{ fails}) = 1 - R_A$

$= P(B \text{ succeeds})$
$\Rightarrow Q_B = P(B \text{ fails}) = 1 - R_B$

$R_S = P(\text{Serial system succeeds})$

$\Rightarrow Q_S = P(\text{Serial system fails})$
$= 1 - R_S$

Assuming components A and B are Independent

Then

P(Serial system succeeds)
= P(A succeeds) * P(B succeeds)

Or, $R_S = R_A * R_B$

And

P(Serial system fails)
= 1 - P(Serial system succeeds)

Or, $Q_S = 1 - R_S$
$= 1 - R_A * R_B$

In general, for Non-Redundant Serial Systems of n Independent Components

$$R_S = \prod_{i=1}^{n} R_i$$

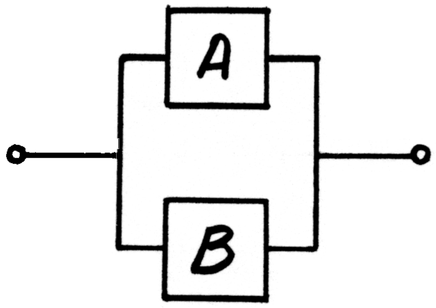
and $$Q_S = 1 - R_S$$
$$= 1 - \prod_{i=1}^{n} R_i$$

Reliability Network Models

Fully Redundant Parallel Systems

System Success
$\Rightarrow$ Success of at least one Component

System Failure
$\Rightarrow$ Failure of all Components



$\Rightarrow A \cup B$

Let $R_A = P(A \text{ succeeds})$
$\Rightarrow Q_A = P(A \text{ fails}) = 1 - R_A$

$R_B = P(B \text{ succeeds})$
$\Rightarrow Q_B = P(B \text{ fails}) = 1 - R_B$

$R_P = P(\text{Parallel system succeeds})$

$\Rightarrow Q_P = P(\text{Parallel system fails})$
$= 1 - R_P$

Assuming components A and B
are Independent

Then

P (Parallel system succeeds)

$= P(A \text{ succeeds}) + P(B \text{ succeeds})$
$- P(A \text{ succeeds}) * P(B \text{ succeeds})$

Or, $R_P = R_A + R_B - R_A * R_B$

And

P (Parallel system fails) = Q_P

And since the Parallel system can fail iff <u>both</u> A <u>and</u> B fail,

Then $Q_P = Q_A * Q_B$

And, so, $R_P = 1 - Q_P = 1 - Q_A * Q_B$

In general, for Fully Redundant Parallel Systems of n Independent Components

$$R_P = 1 - \prod_{i=1}^{n} Q_i$$

and

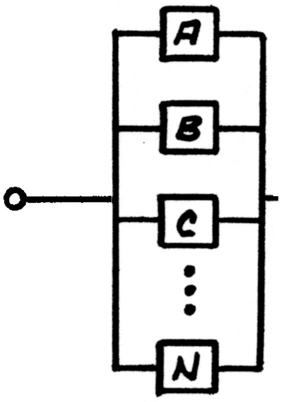
$$Q_P = \prod_{i=1}^{n} Q_i$$

Reliability Network Models

Partially Redundant (k of n) Parallel Systems

System Success
⇒ Success of at least k of n Components

System Failure
⇒ Failure of at least $(n-k+1)$ Components



Assuming independent Component Reliability:

- If all n Components have the same reliability, then the binomial distribution can be applied to determine System reliability

- If Component reliabilities differ, then enumeration is usually required to determine System reliability

For the first case:

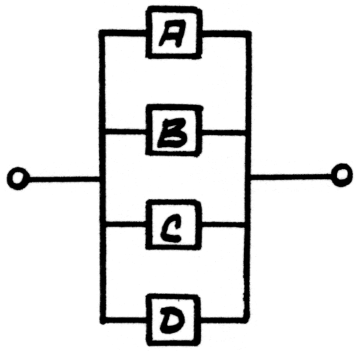
When $R_A = R_B = \cdots = R_N = R$,

Then $$R_P = \sum_{x=k}^{n} \frac{n!}{x!\,(n-x)!} R^x (1-R)^{n-x}$$

(CONTINUED)

For the second case:

As an Example: Consider a parallel system with four components, at least two of which must function for the system to function. Further assume that the probability of success differs for each component (i.e., $R_A \neq R_B \neq R_C \neq R_D$).



System Success ⇒
2 components function
OR 3 components function
OR 4 components function

System Failure ⇒
0 components function
OR 1 component functions

Enumeration of ways system can fail:

COMPONENT	STATUS				
A	S	F	F	F	F
B	F	S	F	F	F
C	F	F	S	F	F
D	F	F	F	S	F

P(System Fails)

$$= R_A Q_B Q_C Q_D + Q_A R_B Q_C Q_D + Q_A Q_B R_C Q_D + Q_A Q_B Q_C R_D + Q_A Q_B Q_C Q_D$$

$$= Q_B Q_C Q_D + Q_A Q_C Q_D + Q_A Q_B Q_D + Q_A Q_B Q_C - 3 Q_A Q_B Q_C Q_D$$

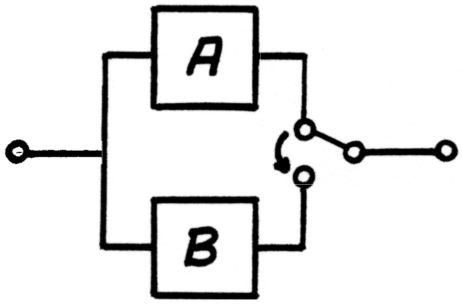
And P(System Succeeds) = 1 − P(System Fails)

Reliability Network Models

Standby Redundant Systems

Systems in which some redundant components operate only when other components fail

Consider a two-component parallel system with a switch:



If switching is "perfect," the probability of switch failure is zero

In the model shown, B is the standby component, and the system switches to B iff A has failed.

Thus P(System fails)

$$= P(A \text{ fails}) * P(B \text{ fails} / A \text{ fails})$$

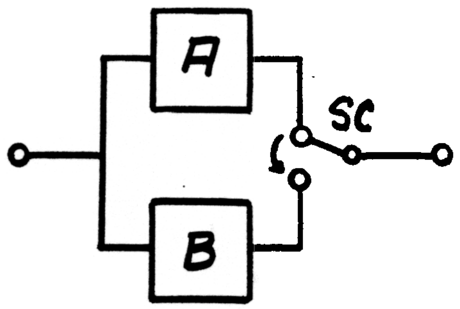
(CONTINUED)

When switching is not "perfect," two types of switch failures can occur:

(1) the switch fails to change over from one component to the other

(2) the switch fails in operating position

Consider only the first type of switch failure:



In what ways can the system fail?

A fails AND SC fails

OR A fails AND SC succeeds AND B fails

Let: $Q_A = P(\text{A fails})$
$Q_B = P(\text{B fails})$
$Q_{SC} = P(\text{SC fails})$
$R_{SC} = P(\text{SC succeeds}) = 1 - Q_{SC}$

Then, P(System Fails)

$$= Q_A * Q_{SC} + Q_A * R_{SC} * Q_B$$
$$= Q_A(1 - R_{SC}) + Q_A * R_{SC} * Q_B$$
$$= Q_A - Q_A R_{SC}(1 - Q_B)$$

{See EQ 4.12}

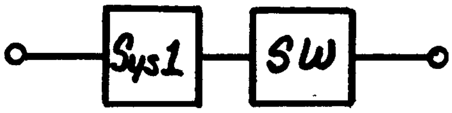
Call this System 1, with
P(System 1 Succeeds) = R_{S1}
AND P(System 1 Fails) = $Q_{S1} = 1 - R_{S1}$

where $Q_{S1} = Q_A - Q_A R_{SC}(1 - Q_B)$

(CONTINUED)

Now that we have accounted for the first type of switch failure, we can incorporate the effects of the switch when it is in operating position

Consider a "new" System 2, which has System 1 in series with the switch in operating position



Now System 2 can fail if

System 1 fails

<u>OR</u> System 1 Succeeds <u>AND</u> SW fails

Let $R_{S1} = P(\text{System 1 Succeeds})$
$Q_{S1} = P(\text{System 1 Fails}) = 1 - R_{S1}$
$Q_{SW} = P(\text{Switch Fails in Operating Position})$
$Q_{S2} = P(\text{System 2 Fails})$

Then

$$Q_{S2} = Q_{S1} + R_{S1} * Q_{SW}$$
$$= Q_{S1} + (1 - Q_{S1}) * Q_{SW}$$
$$= Q_{S1} + Q_{SW} - Q_{S1} * Q_{SW}$$

{This is in conformance with EQ 4.13}

System Reliability and Probability Distributions

Component reliability is usually not constant over time.

Thus component reliability is frequently modeled by a probability distribution for which the random variable is time.

If, at $t=0$, a component (or system) is known to be operating, then its probability of failure at $t=0$ is zero

As $t \to \infty$, however, the probability of failure approaches 1, since it is a certainty that the component (or system) will fail, given that the exposure time to failure is long enough.

This situation is analogous to a cumulative probability density (i.e., probability distribution function), yielding a measure of the probability of failure as a function of time.

(CONTINUED)

In reliability terminology, this measure is known as the cumulative failure distribution, designated by $Q(t)$.

The complement of $Q(t)$ is known as the survivor function, designated by $R(t)$, where $R(t) = 1 - Q(t)$.

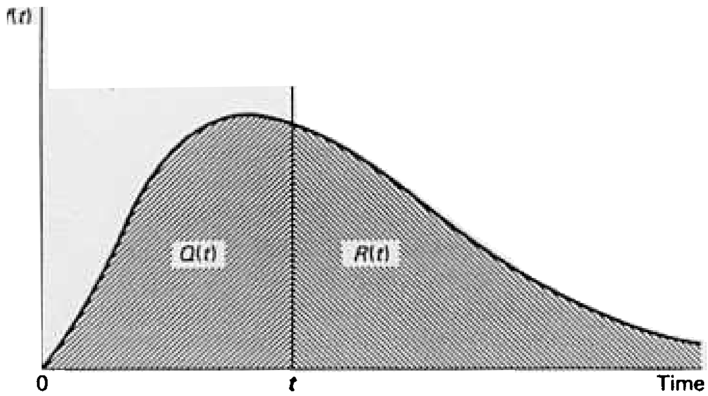


Fig. 6.1 Hypothetical failure density function. $Q(t)$, probability of failure in time t, $R(t)$, probability of surviving beyond time t

A third type of function used in reliability is the transition rate, most frequently referred to as the hazard rate, or hazard function, designated by $\lambda(t)$, where

$$\lambda(t) = \frac{\text{\# failures per unit time}}{\text{\# components exposed to failure}}$$

The hazard function and the survivor function are related in accord with

$$R(t) = \exp\left\{-\int_0^t \lambda(t)\,dt\right\}.$$

(CONTINUED)

With an n-component series system having hazard rate $\lambda(t)$,

$$R_S(t) = \prod_{i=1}^{n} \exp\left[\int_0^t \lambda_i(t)\,dt\right]$$

and with an n-component parallel system having hazard rate $\lambda(t)$,

$$Q_P(t) = \prod_{i=1}^{n} \left\{1 - \exp\left[\int_0^t \lambda_i(t)\,dt\right]\right.$$

Now, a probability distribution used frequently in reliability is the exponential distribution

It has an interesting property referred to as "lack of memory"

This refers to the fact that the hazard rate of the exponential distribution is constant and thus independent of time, so that

$\lambda(t)$ = hazard rate = λ = failure rate

In such case, $R_i(t) = e^{-\lambda_i t}$

$$Q_i(t) = 1 - e^{-\lambda_i t}$$

(CONTINUED)

And so, when the probabilities of failure of system components as a function of time are exponentially distributed,

$$R_S(t) = \exp\left(-\sum_{i=1}^{n} \lambda_i t\right)$$

for a series system

and

$$Q_P(t) = \prod_{i=1}^{n} \left\{1 - \exp(-\lambda_i t)\right\}$$

for a parallel system

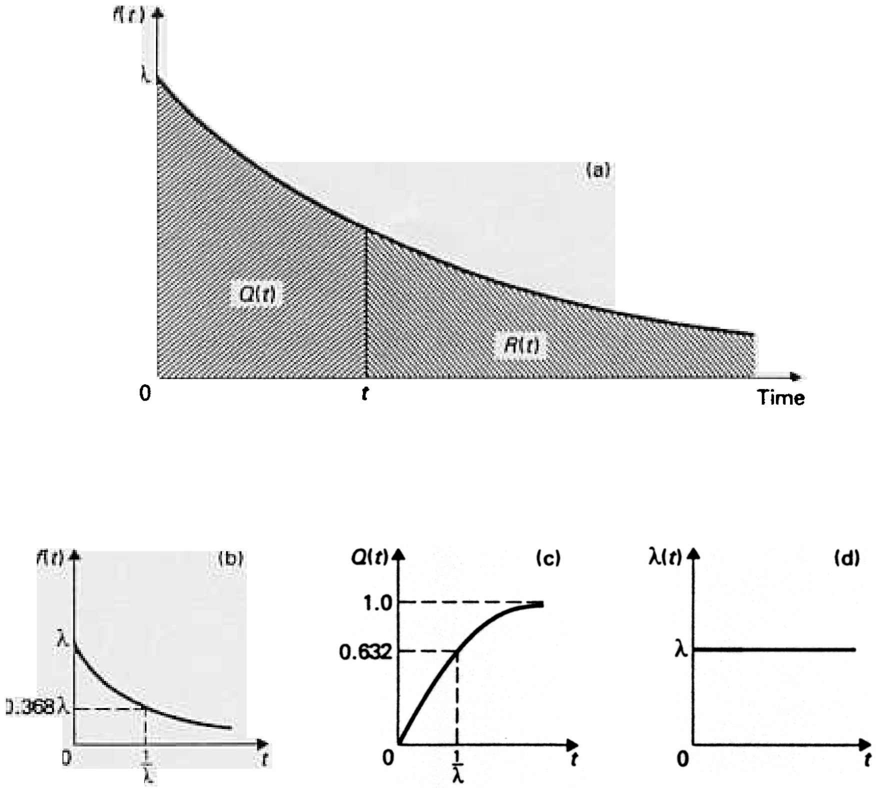


Fig. 6.13 Exponential reliability functions. (a) Areas showing $Q(t)$ and $R(t)$. (b) Failure density function. (c) Cumulative failure distribution. (d) Hazard rate

(CONTINUED)

Note that

If the components in a system all have exponentially distributed reliability functions

Then -

For a serial system, the system reliability is also exponentially distributed

For a parallel system, the system reliability is <u>not</u> exponentially distributed, even though the system reliability function is composed of a series of exponential functions.

Structural Reliability

Consider a single element:

Let L be a continuous random variable describing load

be a continuous random variable describing strength

Then

the probability of failure of the element can be defined by the joint probability density function

$$P_f = P(L \geq S) = \iint f_{S,L}(s,l)$$

over the failure region

$(s - l \leq 0)$

A structural system can usually be classified as one of the following:

- <u>Weakest-link</u> (i.e., series) system — the system fails if any of its elements fail
- <u>Fail-safe</u> (i.e., parallel) system — single element failure does not <u>typically</u> result in system failure

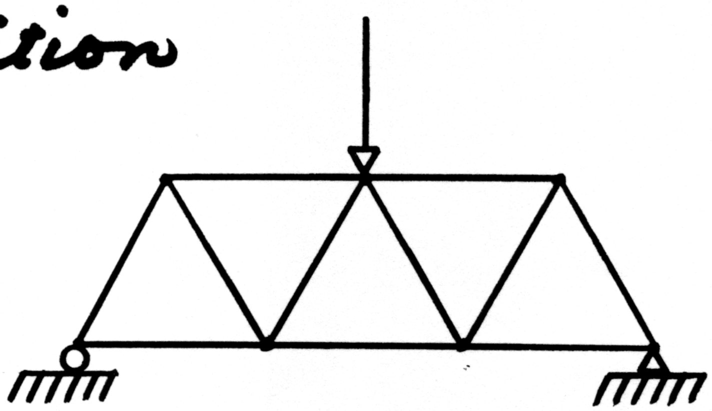
In addition, the specific nature of the failure of a single element may differ.

For example -

- Element made of <u>brittle material</u> loses its load-carrying capacity after failure
- Element made of <u>ductile material</u> does not lose load-carrying capability after failure but cannot carry any additional load after failure

Consider

Statically determinate truss subjected to a single loading condition



This can be idealized as a weakest-link (series) system

Now - Let

$F_{S_i}(s_i)$ = Probability distribution function of the strength of element i

Then

$$F_S(s) = P(S \leq s)$$
$$= 1 - \prod_{i=1}^{n}\left[1 - F_{S_i}(s_i)\right]$$

= Probability distribution function of the strength, S, of the series system

Note that it is assumed here that the strengths, s_i, of the individual elements are independent.

Now - If a load, l, is applied to the system, the force (or stress) induced in element i will be $a_i l$, where

a_i = force (or stress) induced in element i when a unit load is applied to the system.

The value of a_i is determined using standard methods of structural analysis.

Then the Probability of Failure, P_f, of the system may be written as

$$P_f = P(L \geq S)$$

$$= 1 - \int_{l=0}^{\infty} \left[P(S > l) \right] f_L(l)\, dl$$

$$= 1 - \int_{l=0}^{\infty} \left[\prod_{i=1}^{n} \{ 1 - F_{S_i}(a_i l) \} \right] f_L(l)\, dl$$