Repeated Measures ANOVA

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Ψ320

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Major Topics

- What are repeated-measures?
- An example
- Assumptions
- Advantages and disadvantages
- Effect size

Repeated Measures?

- Between-subjects designs
  - different subjects serve in different treatment levels
  - (what we already know)
- Repeated-measures (RM) designs
  - each subject receives all levels of at least one independent variable
  - (what we're learning today)

Repeated Measures

- All subjects get all treatments.
- All subjects receive all levels of the independent variable.
- Different *n*'s are unusual and cause problems (i.e. dropout or mortality).
- Treatments are usually carried out one after the other (in serial).

Example: Counseling For PTSD

- Foa, *et al.* (1991)
  - Provided supportive counseling (and other therapies) to victims of rape
  - Do number of symptoms change with time?
    - There's no control group for comparison
    - Not a test of effectiveness of supportive counseling

Example: Counseling For PTSD

- 9 subjects measured before therapy, after therapy, and 3 months later
- We are ignoring Foa's other treatment conditions.

Example: Counseling For PTSD

- Dependent variable = number of reported symptoms.
- Question: Do number of symptoms decrease over therapy and remain low?
- Data on next slide

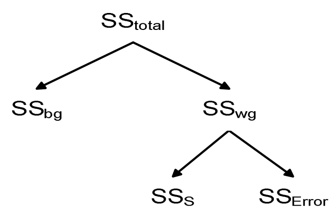
The Data

Patient	Pre	Post	Follow-Up	Subject Mean
1	21	15	15	17.000
2	24	15	8	15.667
3	21	17	22	20.000
4	26	20	15	20.333
5	32	17	16	21.667
6	27	20	17	21.333
7	21	8	8	12.333
8	25	19	15	19.667
9	18	10	3	10.333
Mean	23.889	15.667	13.222	17.593
SD	4.197	4.243	5.783	4.072

Preliminary Observations

- Notice that subjects differ from each other.
  - Between-subjects variability
- Notice that means decrease over time
  - Faster at first, and then more slowly
  - Within-subjects variability

Partitioning Variability



This partitioning is reflected in the summary table.

Sums of Squares

- The total variability can be partitioned into Between Groups (e.g. measures), Subjects and Error Variability

$$\sum\left(Y_i - \bar{Y}_{GM}\right)^2 = \sum n_j\left(\bar{Y}_j - \bar{Y}_{GM}\right)^2 + g\sum\left(\bar{Y}_i - \bar{Y}_{GM}\right)^2 + $$
$$+\left[\sum\left(Y_i - \bar{Y}_j\right)^2 - g\sum\left(\bar{Y}_i - \bar{Y}_{GM}\right)^2\right]$$

$$SS_{Total} = SS_{BetweenGroups} + SS_{Subjects} + SS_{Error}$$

$$SS_T = SS_{BG} + SS_S + SS_{Error}$$

Deviation Sums of Squares

$$SS_{Total} = \sum\left(Y_i - \bar{Y}_{GM}\right)^2 =$$
$$= (\_\_\_ - \_\_\_\_)^2 + (\_\_\_ - \_\_\_\_)^2 + (\_\_\_ - \_\_\_\_)^2 +$$
$$(26 - 17.593)^2 + (32 - 17.593)^2 + (27 - 17.593)^2 +$$
$$(21 - 17.593)^2 + (25 - 17.593)^2 + (18 - 17.593)^2 +$$
$$(15 - 17.593)^2 + (15 - 17.593)^2 + (17 - 17.593)^2 +$$
$$+ \cdots + (8 - 17.593)^2 + (15 - 17.593)^2 + (3 - 17.593)^2 =$$
$$= 1114.519$$

Deviation Sums of Squares

$$SS_{BG} = \sum n_j \left(\bar{Y}_j - \bar{Y}_{GM}\right)^2 =$$
$$= [\_\_ * (\_\_\_\_ - \_\_\_\_)^2] + [\_\_ * (\_\_\_\_ - \_\_\_\_)^2] +$$
$$+ [\_\_ * (13.222 - 17.593)^2] = 562.074$$

Deviation Sums of Squares

$$SS_{Subject} = g \sum \left(\bar{Y}_i - \bar{Y}_{GM}\right)^2 =$$
$$= [\_\_ * (\_\_\_\_ - \_\_\_\_)^2] + [\_\_ * (\_\_\_\_ - \_\_\_\_)^2] +$$
$$+ [\_\_ * (\_\_\_\_ - \_\_\_\_)^2] + [\_\_ * (20.333 - 17.593)^2] +$$
$$+ [\_\_ * (21.667 - 17.593)^2] + [\_\_ * (21.333 - 17.593)^2] +$$
$$+ [\_\_ * (12.333 - 17.593)^2] + [\_\_ * (19.667 - 17.593)^2] +$$
$$+ [\_\_ * (10.333 - 17.593)^2] = 397.852$$

Deviation Sums of Squares

$$SS_{Error} = \left[\sum \left(Y_i - \bar{Y}_j\right)^2 - g \sum \left(\bar{Y}_i - \bar{Y}_{GM}\right)^2\right] =$$
$$\sum \left(Y_i - \bar{Y}_j\right)^2 = (\_\_\_ - \_\_\_\_)^2 + (\_\_\_ - \_\_\_\_)^2 + (\_\_\_ - \_\_\_\_)^2 +$$
$$+ (26 - 23.889)^2 + (32 - 23.889)^2 + (27 - 23.889)^2 +$$
$$+ (21 - 23.889)^2 + (25 - 23.889)^2 + (18 - 23.889)^2 +$$
$$+ (15 - 15.667)^2 + (15 - 15.667)^2 + (17 - 15.667)^2 +$$
$$+ \cdots + (8 - 13.222)^2 + (15 - 13.222)^2 + (3 - 13.222)^2 =$$
$$= 552.444$$
$$= 552.444 - 397.852 = 154.592$$

Computational Approach

Patient	Pre	Post	Follow-Up	Subject Sum
1	21	15	15	51
2	24	15	8	47
3	21	17	22	60
4	26	20	15	61
5	32	17	16	65
6	27	20	17	64
7	21	8	8	37
8	25	19	15	59
9	18	10	3	31
Sum	**215**	**141**	**119**	**475**
ΣY^2				**9471**

Computational Sums of Squares

$$SS_{bg} = \frac{\sum\left(\sum A_j\right)^2}{s} - \frac{T^2}{as}$$

$$SS_{Subject} = \frac{\sum\left(\sum S_i\right)^2}{a} - \frac{T^2}{as}$$

$$SS_{Error} = \sum Y^2 - \frac{\sum\left(\sum A_j\right)^2}{s} - \frac{\sum\left(\sum S_i\right)^2}{a} + \frac{T^2}{as}$$

$$SS_{Total} = \sum Y^2 - \frac{T^2}{as}$$

Computational SS Example

$$SS_{bg} = \frac{\_\_\_^2 + \_\_\_^2 + 119^2}{\_\_} - \frac{\_\_\_^2}{\_\_(\_\_)} = \frac{\_\_\_\_\_}{\_\_} - \frac{\_\_\_\_\_}{\_\_} =$$

$$= \_\_\_\_\_\_\_ - 8356.481 = 562.075$$

$$SS_S = \frac{\_\_\_^2 + \_\_\_^2 + \_\_\_^2 + 61^2 + 65^2 + 64^2 + 37^2 + 59^2 + 31^2}{\_\_} - 8356.481 =$$

$$= \frac{\_\_\_\_\_}{\_\_} - 8356.481 = \_\_\_\_\_\_\_ - 8356.481 = 397.852$$

$$SS_{Error} = 9471 - 8918.556 - 8754.333 + 8356.481 = 154.592$$

$$SS_T = 9471 - 8356.481 = 1114.519$$

Degrees of Freedom

From BG ANOVA:

$$df_{bg} = g - 1 = 3 - 1 = 2$$

$$df_{ws} = g(n-1) = N - g = 27 - 3 = 24$$

$$\rightarrow df_s = n - 1 = 9 - 1 = 8$$

$$\rightarrow df_{error} = (n-1)*(g-1) = 8*2 = 16$$

$$df_{total} = N - 1 = 27 - 1 = 26$$

Summary Table

Source	SS	df	MS	F
Time	**562.074**	**2**	**281.037**	**29.087**
WS (from BG design)	*552.444*	*24*		
Subject	397.852	8	49.731	
Error	**154.593**	**16**	**9.662**	
Total	1114.519	26		

- F_{crit} with 2 and 16 degrees of freedom is 3.63 we would reject h_0

Plot of the Data

Interpretation

- Note parallel with diagram
- Note subject differences not in error term
- Note MS_{error} is denominator for *F* on Time
- Note SS_{time} measures what we are interested in studying

Assumptions

- Correlations between trials are all equal
  - Actually more than necessary, but close
  - Matrix shown below

	Pre	Post	Followup
Pre	1.00	.637	.434
Post		1.00	.742
Followup			1.00

Assumptions

- Previous matrix might look like we violated assumptions
  - Only 9 subjects
  - Minor violations are not too serious.
- Greenhouse and Geisser (1959) correction (among many)
  - Adjusts degrees of freedom

Multiple Comparisons

- With few means:
  - *t* test with Bonferroni corrections
  - Limit to important comparisons
- With more means:
  - Require specialized techniques
    - Trend analysis

Advantages of RM Designs

- Eliminate subject differences from error term
  - Greater power
- Fewer subjects needed
- Often only way to address the problem
  - This example illustrates that case.

Disadvantages of RM designs

- Carry-over effects
  - Counter-balancing
- May tip off subjects to what you are testing
- Some phenomenon cannot be tested in a repeated measure fashion (e.g. anything that requires tricking the participant)

Effect Size

- Simple extension of what we said for *t* test for related samples.
- Stick to pairs of means.
- OR
  - η^2 can be used for repeated measures data as well
  - Some adjustments can make it more meaningful