


Probability


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Probability

- Part of everyday life
 - probability in poker
 - probability in lottery
 - probability of rain
- Guides our expectations
- Lies at the heart of statistical inference
- Whenever we generalize from a sample to a population, we do so with a probability statement

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Definition of Probability

- Theoretical Definition - Analytic View
 - used if we can list **all** possible outcomes of an event and if each outcome is equally likely.
- The probability of event A is:

$$p(a) = \frac{\text{Number of outcomes favorable to A}}{\text{Total Number of Possible Outcomes}}$$
- Example: What is the probability of rolling a 4 with 2 6-sided dice?

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Possible Outcomes for 2 Fair Dice

| | | | | | | | |
|-------|-------|---|---|---|----|----|----|
| Die 1 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 1 | 2 | 3 | 4 | 5 | 6 | |
| | Die 2 | | | | | | |

There are 3 ways to roll a 4 with 2 dice: $p(4) = 3/36 = 1/12$

Definition of Probability

- Empirical definition of probability - Relative Frequency View
 - Used when you don't know all the possible outcomes or they are not equally likely.
 - In this case, we define probability empirically, by observation:

$$p(a) = \frac{\text{Number of times A occurs}}{\text{Large number of trials}}$$

- A much more mechanical approach to probability
- Roll dice 100 times and count how many 4s

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More Definitions

- Event – the outcome of a trial or probabilistic experiment
 - e.g. Getting a King in a deck of 52 cards
- Mutually Exclusive - events that cannot happen at the same time
 - $P(A \text{ and } B) = 0$
 - e.g. $P(\text{Male and Female}) = 0$
- Exhaustive – A set of events that represents all possible outcomes
 - e.g. $P(\text{Male or Female}) = 1$

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More Definitions

- Independent Events – one event does not have an impact on the probability of the occurrence of another
- Sampling with replacement – the result of any event is replaced before the next event
 - Draw a card out of a deck of 52 and put it back before drawing again
 - e.g. $P(\text{King of Hearts}) = 1/52$ for every draw



One more definition

- Sampling without replacement - the result of any event is not replaced before the next event
 - Draw a card out of a deck of 52 and leave it out before drawing again
 - e.g. $P(\text{King of Hearts}) = 1/52$ for first draw and $P(\text{King of Hearts}) = 1/51$ for second, etc.
 - e.g. Raffle Drawings



Probabilities and Proportions

- By either definition (theoretical or empirical), probabilities are proportions
- In inference, probabilities are often treated as expected proportions
 - We would expect half the number of coin flips to be heads
 - We expect 51% of marriages to end in divorce



Probabilities and Proportions

- Probabilities range between zero and one (like any proportion)
- Probability of an impossible event is 0
 - $P(3 \text{ inches of snow in LA}) = 0$
- Probability of a "sure thing" is 1.
 - $P(\text{Smog in LA}) = 1$



Probabilities and Proportions

- Probability of an event **not** happening = $1 -$ the probability of the event happening
 - $P(\text{not Ace}) = 1 - P(\text{Ace})$
 - $P(\text{not Ace}) = 1 - 4/52$ or $1/13$
 - $P(\text{not Ace}) = 1 - 1/13 = 12/13$



Laws of Probability

- Additive Law of Probability –
 - Given mutually exclusive events:
 $P(A \text{ or } B) = P(A) + P(B)$
 - See OR, think ADD
- Multiplicative Law of Probability –
 - Given independent events
 $P(A \text{ and } B) = P(A) \times P(B)$
 - See AND, think MULTIPLY
 - This is called the joint probability
 - If the events are dependent this gets more complicated



Examples

- In a normal deck of 52 cards what is the probability of :
 - Drawing an ace?
 - Drawing a spade?
 - Not drawing a spade?
 - Drawing the ace of spades?
 - Drawing an ace and a spade?
 - Drawing an ace or a spade?
 - Drawing 2 aces in a row?
 - Drawing 2 spades in a row?
 - Drawing a spade given you've drawn a black card?

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Conditional Probability

- This is the probability that an event will occur, given that some other event has occurred.
 - $P(A | B) = P(\text{Spade} | \text{Black Suit}) = ???$
 - i.e. Knowing you selected a black suit, what is the probability of a spade?
 - The conditional property usually works to reduce the total number of events
 - $P(\text{Red Suit} | \text{Ace}) = ??$
 - $P(\text{Ace} | \text{King}) = ??$

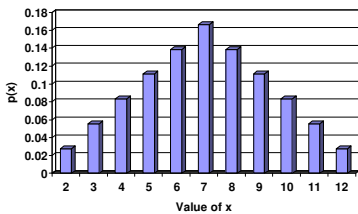
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Probability Distributions for Discrete Variables

| X | p |
|----|------|
| 12 | 1/36 |
| 11 | 2/36 |
| 10 | 3/36 |
| 9 | 4/36 |
| 8 | 5/36 |
| 7 | 6/36 |
| 6 | 5/36 |
| 5 | 4/36 |
| 4 | 3/36 |
| 3 | 2/36 |
| 2 | 1/36 |

Probability Distribution for the Possible Events when Rolling 2 Dice



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| 5 | 4/36 |
| 4 | 3/36 |
| 3 | 2/36 |
| 2 | 1/36 |

Thus, the probability that x falls in the interval between any two numbers a and b, inclusive, is found by simply summing the probabilities for x over all possible values between a and b, inclusive.

$p(a \leq x \leq b)$ = sum of $p(x = c)$ for all values c such that: $a \leq c \leq b$.

For example, the probability that x is between 3 and 5, inclusive is:

$$p(3 \leq x \leq 5) = p(3) + p(4) + p(5) = 2/36 + 3/36 + 4/36 = 1/4 = .25.$$

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Probability Distributions for Discrete Variables

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|----|------|
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| 7 | 6/36 |
| 6 | 5/36 |
| 5 | 4/36 |
| 4 | 3/36 |
| 3 | 2/36 |
| 2 | 1/36 |

By the same argument,

$P(x \leq a)$ = sum of $p(x = c)$ for all c such that $c \leq a$.

For example, $p(x \leq 4) = p(x = 4) + p(x = 3) + p(x = 2) = 3/36 + 2/36 + 1/36 = 1/6 = .1667.$

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Probability Density Functions for Continuous Variables

- This was covered in the last section but to review:

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} (e)^{-(X-\mu)^2/2\sigma^2}$$

- Relies on Mean and SD
- Allows us to calculate probability for a range of scores

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Density over a range =
Area = Probability

- Mean of 30, SD 10
- Probability of scores falling below 27
- Probability = .2743 (from Z table)

