What is a typical score like?

There are three indices of this central tendency:
- Mode
- Median
- Mean

<table>
<thead>
<tr>
<th>Measure</th>
<th>Definition</th>
<th>Level of Measurement</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Most frequent value</td>
<td>nom., ord., int./rat.</td>
<td>Crude</td>
</tr>
<tr>
<td>Median</td>
<td>Middle value</td>
<td>ord., int./rat.</td>
<td>Only two points contribute</td>
</tr>
<tr>
<td>Mean</td>
<td>Arithmetic average</td>
<td>int./rat.</td>
<td>Affected by skew</td>
</tr>
</tbody>
</table>
The Mode

- Mode: most common (frequent) number
- Peak of distribution
- Always an observed score
- With grouped data is merely the *midpoint* of the most frequent measurement class.
- Thus, if a case were drawn at random from the distribution, that case is more likely to fall in the modal class than any other.

**Important interpretation**

How to find the mode?

![Diagram of one and two modes](diagram)

Problems with the Mode

- However, there are three disadvantages with the mode:
  1. Multiple modes (especially with grouped data)
  2. The mode is very sensitive to the size and number of class intervals (different intervals = different modes)
  3. The mode of a sample undependable when estimated population
The Median
- Median: middle of distribution (50th percentile)
- Not applicable to nominal data; ordinal, interval & ratio data only
- Is considerably less sensitive when grouping into class intervals.
- More useful for making inferences (although not the best)
- Not affected by outliers.

Median

- The median requires 1 or 2 pieces of information
- If there are an odd number of scores the median is the center score
- If there is an even number the median is midpoint between the two middle numbers
Median

- **Examples**
  - **Data Set 1:** \{7, 2, 9, 3, 4, 5, 8\}
    - Ordered: __________
    - **Median** = _______
  - **Data Set 2:** \{11, 15, 10, 9, 5, 13, 12\}
    - Ordered: __________
    - **Median** = _______

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Median

**Data set 1:** \{12, 9, 8, 5, 3, 2\}
**Data set 2:** \{77, 50, 8, 5, 5, 5\}

- Example: In a six-score set of data the only scores that matter when calculating the median are the two middle scores
- The rest of the data is ignored
- Because of this the median is not sensitive to outlying scores

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Median

- The average **absolute deviation** is smallest when taken from the median.

\[
A.A.D = \frac{\sum |X_i - Mdn|}{n}
\]
The Mean

- Only used for interval & ratio data.

\[ \text{Mean} = M_X = \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \]

- Major advantages:
  - The sample value is a very good estimate of the population value.
  - Mean of the sample = \( \bar{X} \)
  - Mean of the population = \( \mu \)

Example

- Data Set: \{3, 4, 6, 10\}

- Formula: \[ \bar{X} = \frac{\sum X}{n} \]

\[ \bar{X} = \frac{3 + 4 + 6 + 10}{4} = \]

The Mean and Deviations

- For the mean the following is always true.

\[ \sum (X_i - \bar{X}) = 0 \]

\[ \sum (X_i - \bar{X})^2 = \text{minimum}^* \]

*When compared to the other 2 measures of central tendency
Mean: Demonstration

\[ \text{Score} \quad (X_i - \bar{X}) \quad (X_i - \bar{X})^2 \]

\[ \begin{array}{ccc}
3 & (3 - 5.75) = -2.75 & 7.563 \\
4 & (4 - 5.75) = -1.75 & 3.063 \\
6 & (6 - 5.75) = 0.25 & 0.063 \\
10 & (10 - 5.75) = 4.25 & 18.063 \\
\Sigma & 0 & 28.75 \\
\end{array} \]

Since \( \bar{X} = 5.75 \)

The Mean

**Mental Note:** If the mean is guessed as the value of any case drawn at random from a distribution, on average the amount of signed error will be 0.

- This is a most important interpretation of the mean and why it’s so most often!

Disadvantage

- Highly affected by outliers

The Influence of Outliers

<table>
<thead>
<tr>
<th>Data</th>
<th>Ordered Data</th>
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<tbody>
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<td>33</td>
</tr>
</tbody>
</table>

Mode = Mode =

Median = Median =

Mean = Mean =
Mode, Mean and Median

Positively Skewed Distribution

Mode Median Mean

Negatively Skewed Distribution

Mean Median Mode

Mode = median = mean?

When the distribution is symmetric

Mode Median Mean
The mean vs. the median

☐ The degree of discrepancy between them indicates the skewness of the data
☐ The closer the two values are the more symmetric the data