Binary Black Holes Simulations and the Hunt for Gravitational Waves

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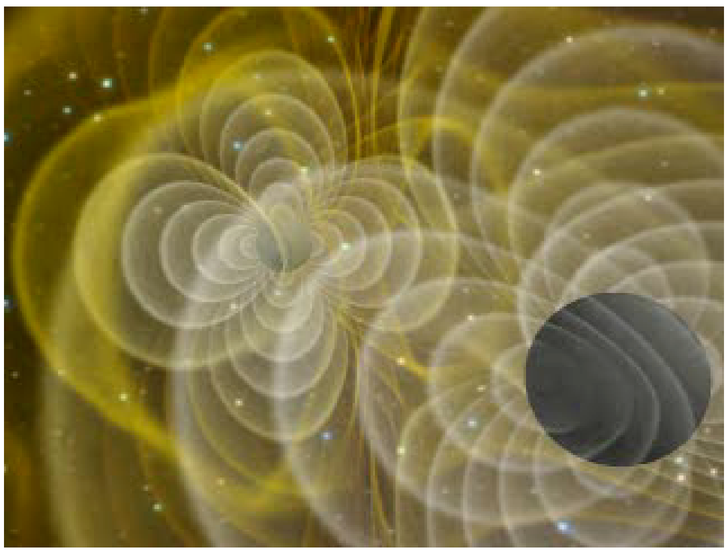


NASA-GSFC

LIGO-LA

An Example of Current Simulations

Herrmann, Hinder, Knapp, Laguna & Shoemaker

PENNSTATE

What made this possible?

- A “good” set of equations
- A “good” set of coordinate
- Adaptive Mesh Refinement
- “Creative Engineering”

“Good” set of equations

Einstein Equations

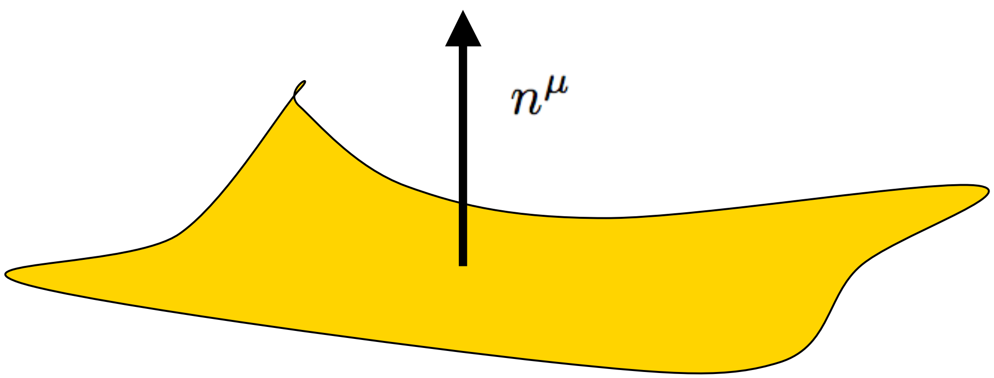
$$G_{\mu\nu} = 8\,\pi\,T_{\mu\nu}$$

Geometry = Matter

$$G_{\mu\nu}(g_{\alpha\beta},\, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta})$$

$g_{\alpha\beta}$: space-time metric

3+1 Decomposition



$n^\mu \perp_\mu^\nu = 0$ $\quad$ $n^\mu n_\mu = -1$

$$\begin{aligned} n^\mu n^\nu\, G_{\mu\nu} &= \ldots \\ n^\mu \perp_\alpha^\nu\, G_{\mu\nu} &= \ldots \\ \perp_\alpha^\mu \perp_\beta^\nu\, G_{\mu\nu} &= \ldots \end{aligned}$$

3+1 Decomposition

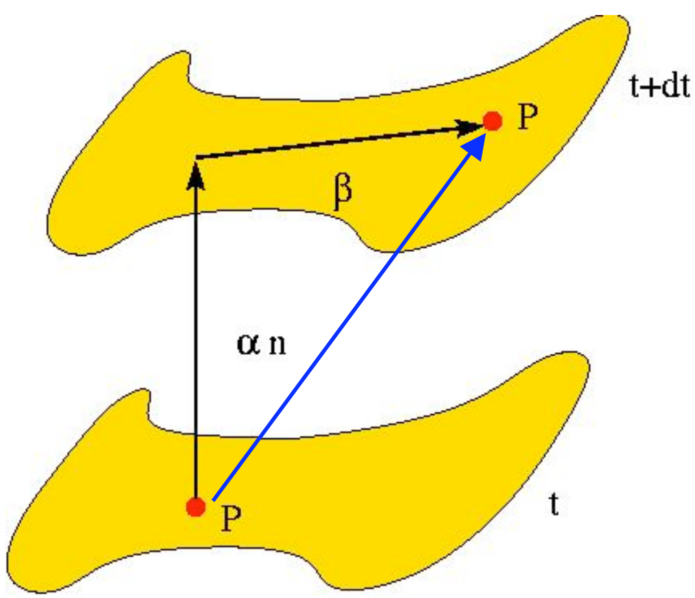
Space-time Metric:

$g_{\mu\nu}$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\mu$$ Spatial Metric

α Lapse Function

β^μ Shift Vector



Second Fundamental Form or Extrinsic Curvature:

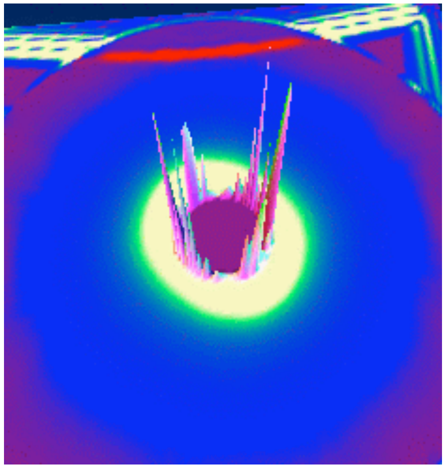
$$K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_n g_{\mu\nu} = -\perp \nabla_{(\mu} n_{\nu)}$$

Standard ADM Equations

$$\begin{aligned}\partial_o \gamma_{ij} &= -2\,\alpha\,K_{ij} \\ \partial_o K_{ij} &= -\nabla_i \nabla_j \alpha + \alpha\,(R_{ij} - 2\,K_{ik} K^k{}_j + K\,K_{ij})\end{aligned}$$

R_{ij} : Ricci tensor $\qquad \partial_o \equiv \partial_t - \mathcal{L}_\beta$

Not a good system!
Only weakly hyperbolic.



Single BH Evolution

BSSN Equations

Baumgarte & Shapiro PRD 59, 024007 (1999)
Shibata & Nakamura PRD 52, 5428 (1995)

$$\gamma_{ij}$$

$$K_{ij}$$

$$\left\{ \begin{aligned} \Phi &= \frac{1}{12} \ln \gamma \\ \hat{\gamma}_{ij} &= e^{-4\,\Phi}\,\gamma_{ij} \\ K &= K^i{}_i \\ \hat{A}_{ij} &= e^{-4\,\Phi}(K_{ij} - \frac{1}{3}\gamma_{ij} K) \\ \widehat{\Gamma}^i &= \hat{\gamma}^{jk}\widehat{\Gamma}^i{}_{jk} \end{aligned} \right.$$

Notice: $\sqrt{\hat{\gamma}} = 1$

BSSN Equations

$$\partial_o \gamma_{ij} = -2\,\alpha\,K_{ij}$$

$$\begin{aligned} \partial_o \Phi &= -\frac{1}{6}\alpha\,K \\ \partial_o \hat{\gamma}_{ij} &= -2\,\alpha\,\hat{A}_{ij} \end{aligned}$$

BSSN Equations

$$\partial_o K_{ij} = -\nabla_i \nabla_j \alpha + \alpha(R_{ij} - 2\,K_{ik}K^k{}_j + K_{ij}K)$$

$$\begin{aligned}
\partial_o K &= -\nabla_i \nabla^i \alpha + \alpha \left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}\hat{K}^2 \right) \\
\partial_o \hat{A}_{ij} &= e^{-4\Phi}[-\nabla_i \nabla_j \alpha + \alpha R_{ij}]^{TF} + \alpha\,(K\,\hat{A}_{ij} - 2\,\hat{A}_{ik}\hat{A}^k{}_j)
\end{aligned}$$

BSSN Equations

$$\sqrt{\hat{\gamma}} = 1$$

$$\widehat{\Gamma}^i \equiv \gamma^{jk} \widehat{\Gamma}^i{}_{jk} = -\partial_j \hat{\gamma}^{ij}$$

$$\begin{aligned}
\partial_o \widehat{\Gamma}^i &= \hat{\gamma}^{jk} \partial_{jk} \beta^i + \frac{1}{3} \hat{\gamma}^{ij} \partial_{jk} \beta^k - 2\,\hat{A}^{ij} \partial_j \alpha \\
&+ 2\,\alpha\,\widehat{\Gamma}^i{}_{jk} \hat{A}^{jk} + 12\,\alpha\,\hat{A}^{ij}\,\partial_j \Phi - \frac{4}{3} \alpha\,\hat{\gamma}^{ij} \partial_j K
\end{aligned}$$

BSSN Equations

$$\partial_o \Phi = -\frac{1}{6}\alpha K$$

$$\partial_o \hat{\gamma}_{ij} = -2\,\alpha\,\hat{A}_{ij}$$

$$\partial_o K = -\nabla_i \nabla^i \alpha + \alpha \left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}\hat{K}^2 \right)$$

$$\partial_o \hat{A}_{ij} = e^{-4\Phi}[-\nabla_i \nabla_j \alpha + \alpha R_{ij}]^{TF} + \alpha\,(K\,\hat{A}_{ij} - 2\,\hat{A}_{ik}\hat{A}^k{}_j)$$

$$\partial_o \hat{\Gamma}^i = \hat{\gamma}^{jk}\partial_{jk}\beta^i + \frac{1}{3}\hat{\gamma}^{ij}\partial_{jk}\beta^k - 2\,\hat{A}^{ij}\partial_j\alpha$$
$$+ \; 2\,\alpha\,\hat{\Gamma}^i{}_{jk}\hat{A}^{jk} + 12\,\alpha\,\hat{A}^{ij}\,\partial_j\Phi - \frac{4}{3}\alpha\,\hat{\gamma}^{ij}\partial_j K$$

BSSN Constraints

$$\sqrt{\hat{\gamma}} = 1 \qquad (1)$$

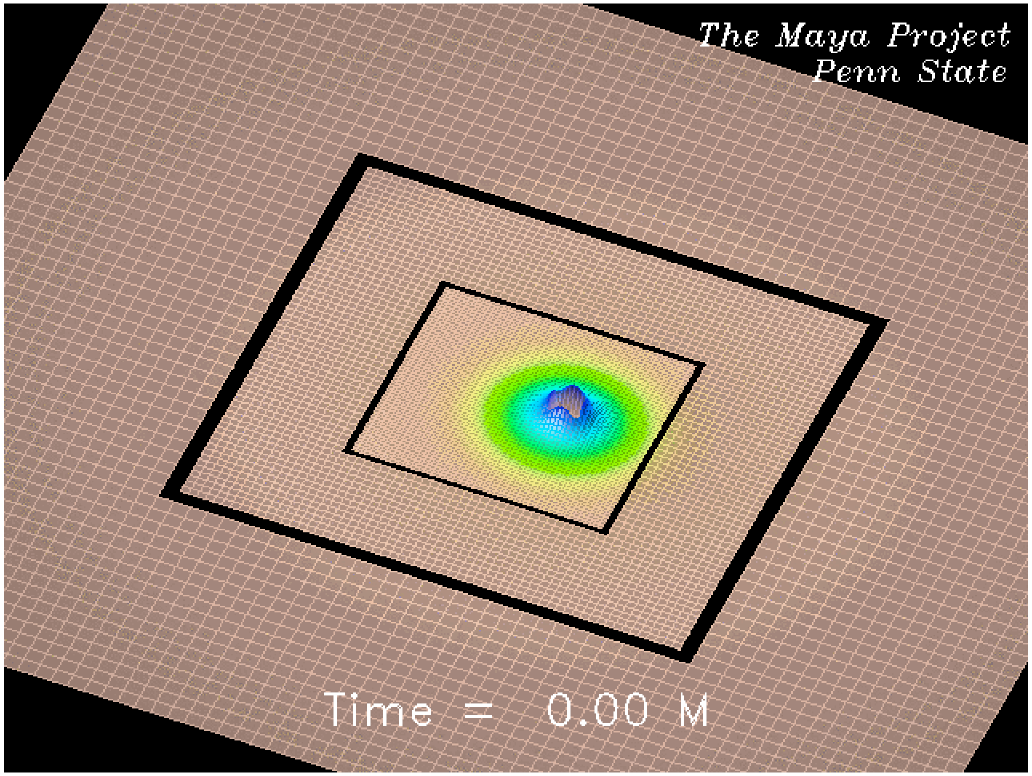
$$\hat{A}^i{}_i = 0 \qquad (2)$$

$$\widehat{\Gamma}^i = \hat{\gamma}^{jk}\widehat{\Gamma}^i{}_{jk} \qquad (3)$$

One evolves all components of $\hat{\gamma}_{ij}$ and $\hat{A}_{ij}$. After each evolution step, $\hat{\gamma}_{ij}$ and $\hat{A}_{ij}$ are adjusted to satisfy (1) and (2).

For $\widehat{\Gamma}^i$, after each evolution step, $\widehat{\Gamma}^i$ is recomputed from (3) and used only on those terms that do not involve derivatives.

Does it work?

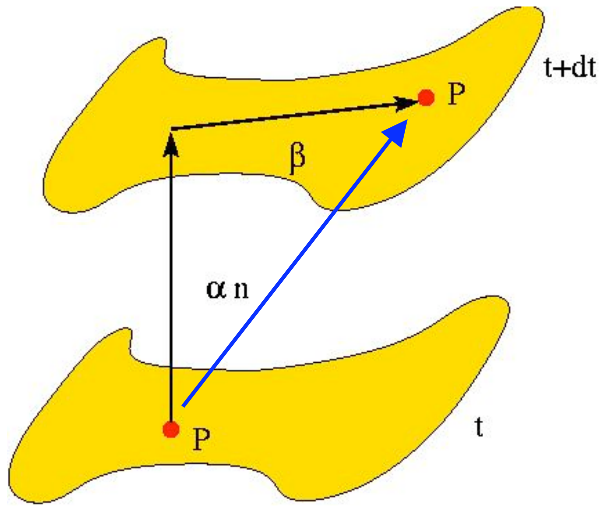


Wobbling BH

Some Facts about the BSSN Equations

- They were found mostly by experimentation and "creative engineering."
- Recently, it was shown that they are strongly hyperbolic for some choices of the lapse and shift.
- For arbitrary lapse and shift, they do not yield stable evolutions.
- No need of introducing parameters.
- Currently the most popular system of equations.

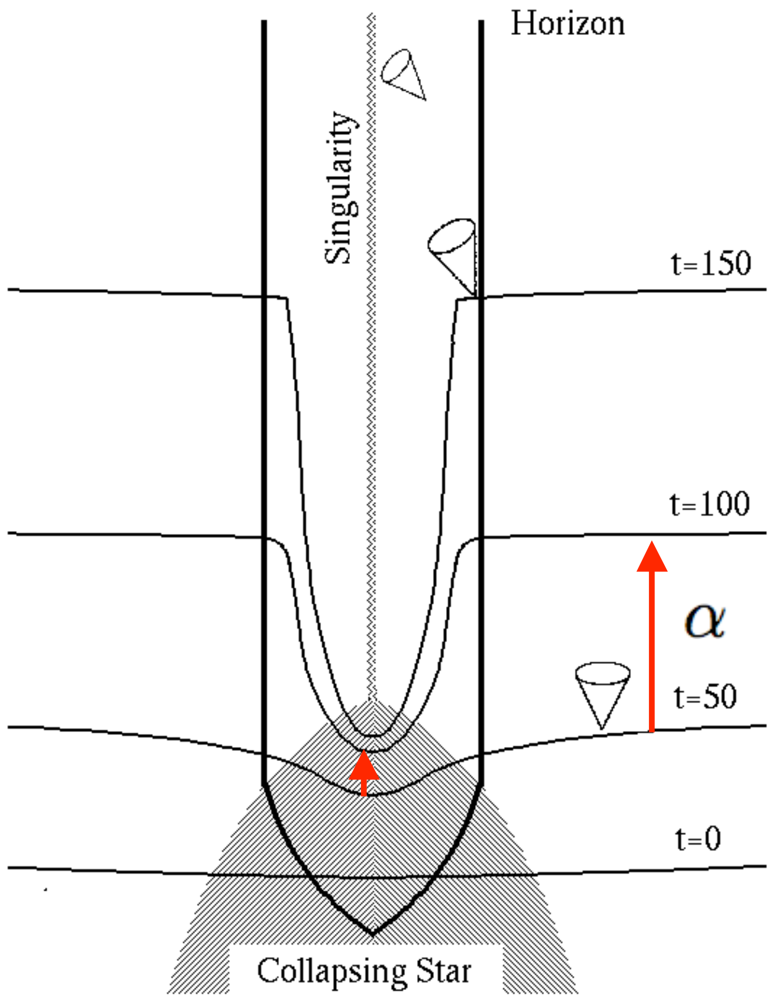
"Good" Coordinate Conditions



Why not?

$$\alpha = 1, \;\; \beta^i = 0$$

Use the gauge freedom of choosing the lapse function α and the shift vector β^i to handle the singularity as well as to cover most of the space-time of interest.



Handling BH Singularities

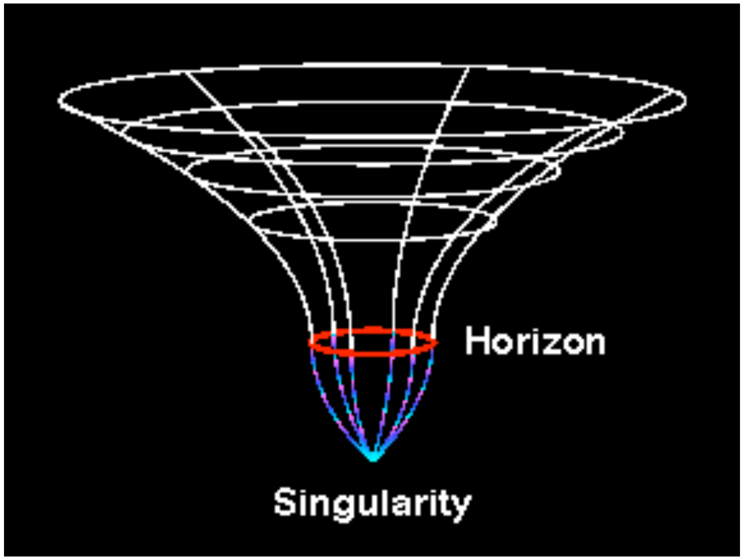
- Excision: Remove the singularity from the computational domain.

- Puncture: Avoid getting close to the black hole singularity.

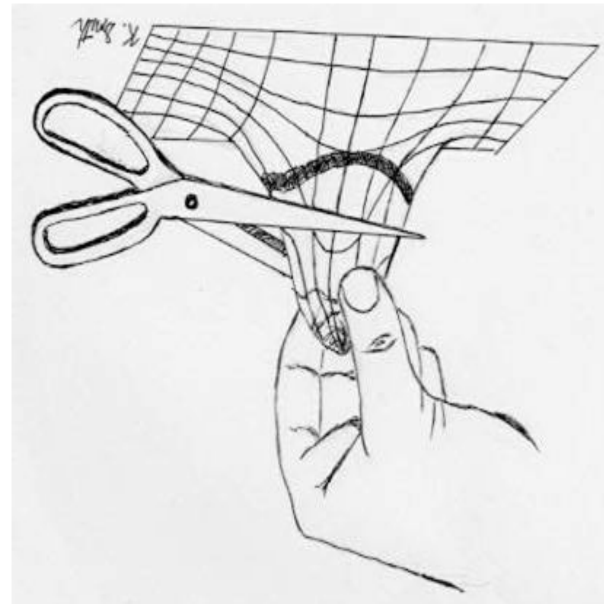
In both cases, one has to be careful of not affecting the physics we are after.

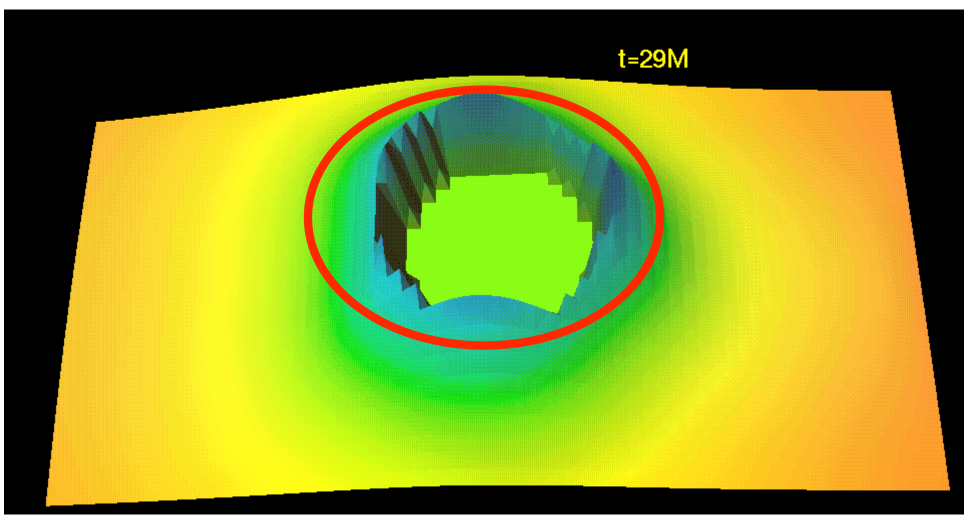
Fortunately, it is the presence of a horizon what saves the day.

Black Hole Singularity: Excision Approach



A. Hamilton





- One must find and track the BH horizon
- Special discretization at the excision boundary

Black Hole Singularity: Puncture Approach

$$ds^2 = -(\alpha^2 + \beta^2)dt^2 + 2\beta_i dx^i dt + e^{-4\Phi}\hat{\gamma}_{ij} dx^i dx^j$$

$$ds^2 = -\left(1 - \frac{2\,M}{R}\right) dt^2 + \frac{dR^2}{(1 - \frac{2\,M}{R})} + R^2\, d\Omega^2$$

Schwarzschild Coords.
BH horizon @ R = 2M

$$ds^2 = -\left(\frac{1 - \frac{M}{2\,r}}{1 + \frac{M}{2\,r}}\right)^2 dt^2 + \left(1 + \frac{M}{2\,r}\right)^4 (dr^2 + r^2\, d\Omega^2)$$

Isotropic Coords.
BH horizon @ r = M/2

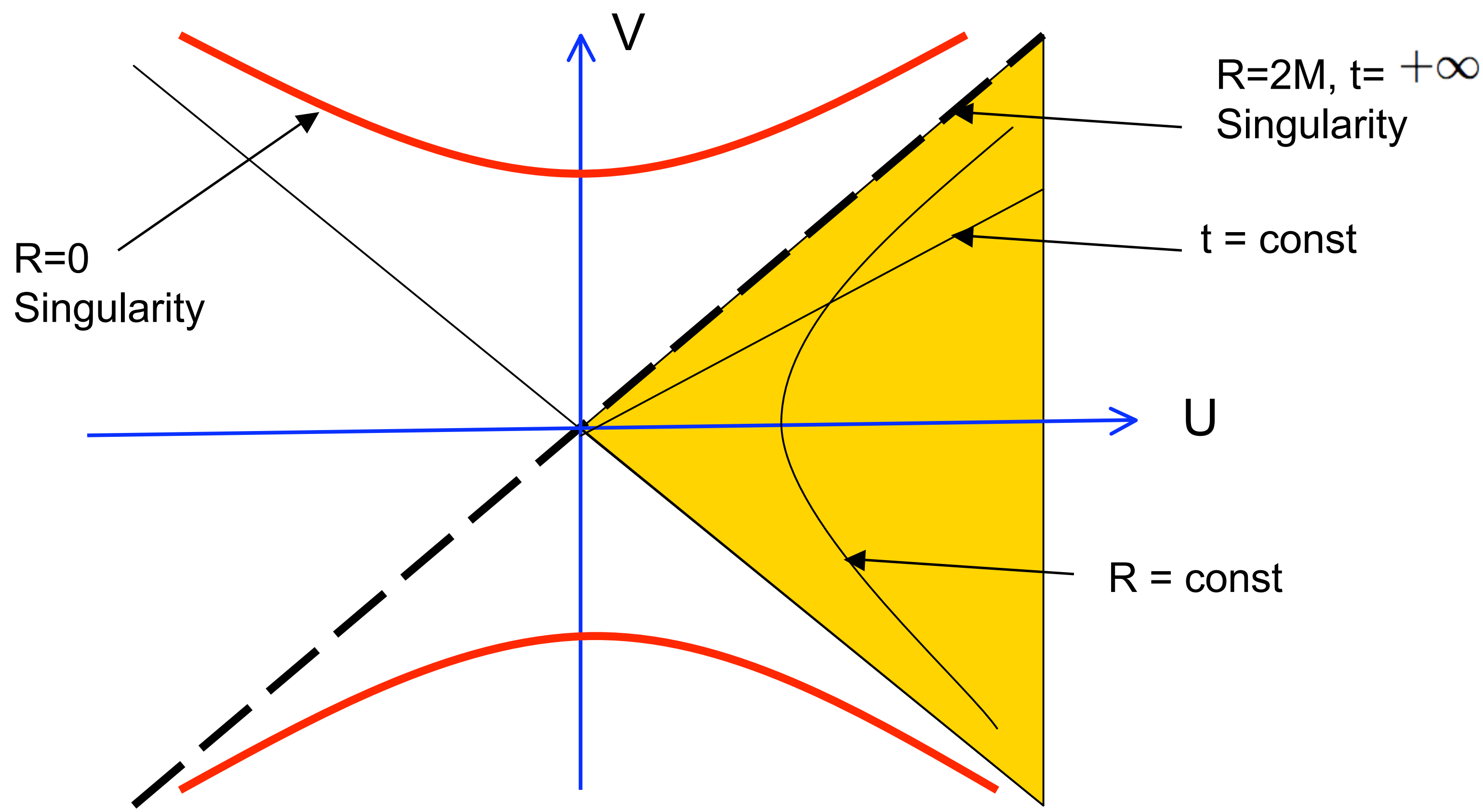
BSSN variables for the isotropic coordinates case:

$$\begin{aligned} e^{\Phi} &= 1 + \frac{M}{2\,r} \\ \hat{\gamma}_{ij} &= \eta_{ij} \\ \alpha &= \frac{1 - \frac{M}{2\,r}}{1 + \frac{M}{2\,r}} \\ \beta^i &= 0 \end{aligned}$$

Kruskal-Szekeres Coordinates

$$ds^2 \quad = \quad \frac{32\,M^3}{R} e^{-R/2M}(-dV^2 + dU^2) + R^2\,d\Omega^2$$

$$\left(\frac{R}{2\,M} - 1\right) e^{R/2M} = U^2 - V^2$$



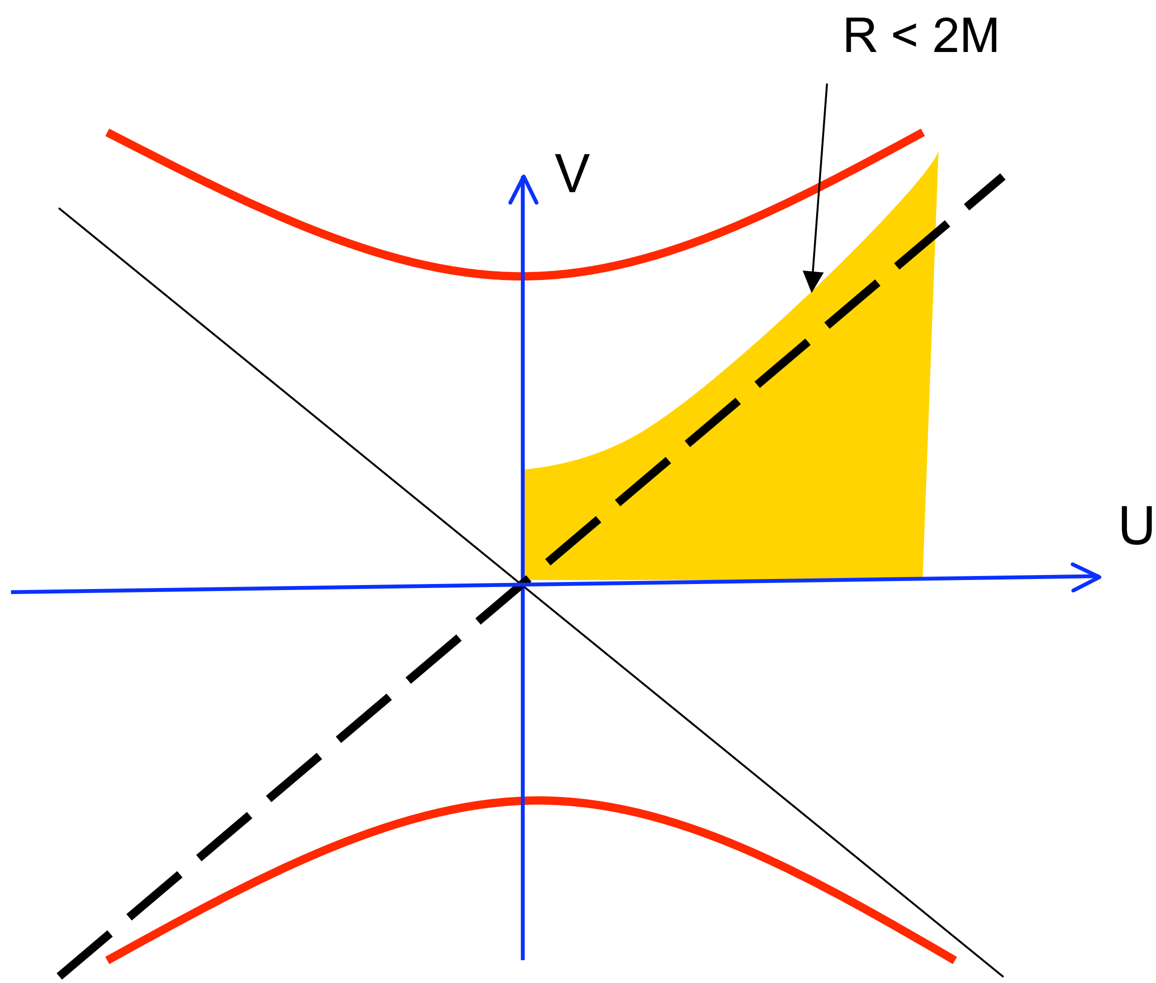
BH Punctures

@ t=0

$$
\begin{aligned}
e^{\Phi} &= 1 + \frac{M}{2\,r} \\
\hat{\gamma}_{ij} &= \eta_{ij} \\
\alpha &= 1 \\
\beta^i &= 0
\end{aligned}
$$

Evolve using:

$$
\begin{aligned}
\partial_t \alpha &= -2\,\alpha\,K + \beta^i \partial_i \alpha \\
\partial_t^2 \beta^i &= \frac{3}{4} \tilde{\Gamma}^i - \eta\,\partial_t \beta^i
\end{aligned}
$$

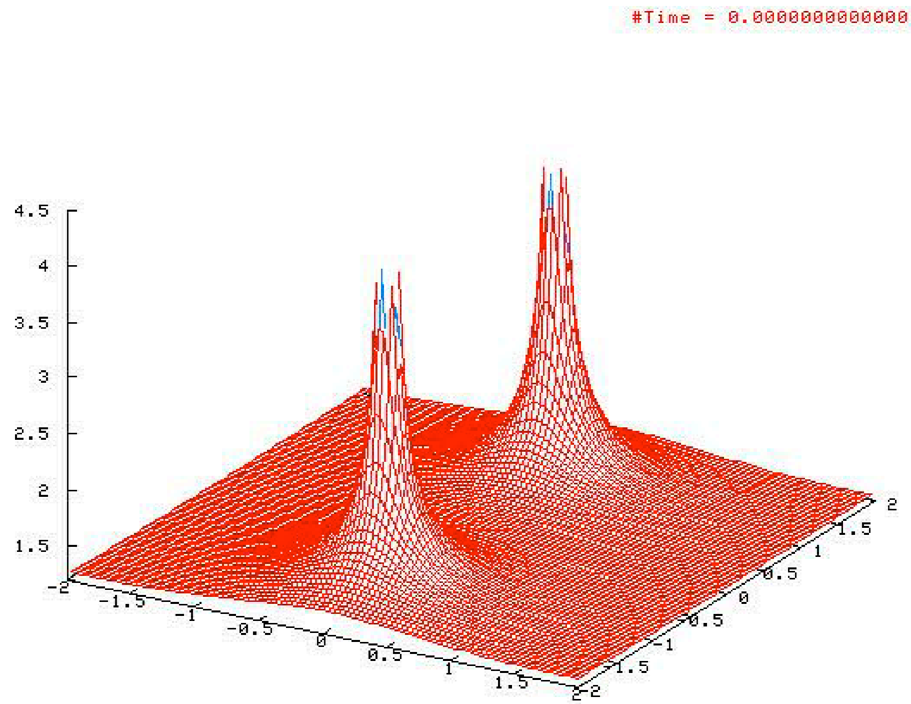


Hannam et al gr-qc/0612097

Multiple Black Hole Punctures

$$e^{\Phi} = 1 + \frac{M}{2\,|\vec{r} - \vec{r}_1|} + \frac{M}{2\,|\vec{r} - \vec{r}_2|}$$

$$\hat{\gamma}_{ij} = \eta_{ij}$$



$$\partial_t \alpha = -2\,\alpha\,K + \beta^i \partial_i \alpha$$

$$\partial_t^2 \beta^i = \frac{3}{4}\tilde{\Gamma}^i - \eta\,\partial_t \beta^i$$

Old View:

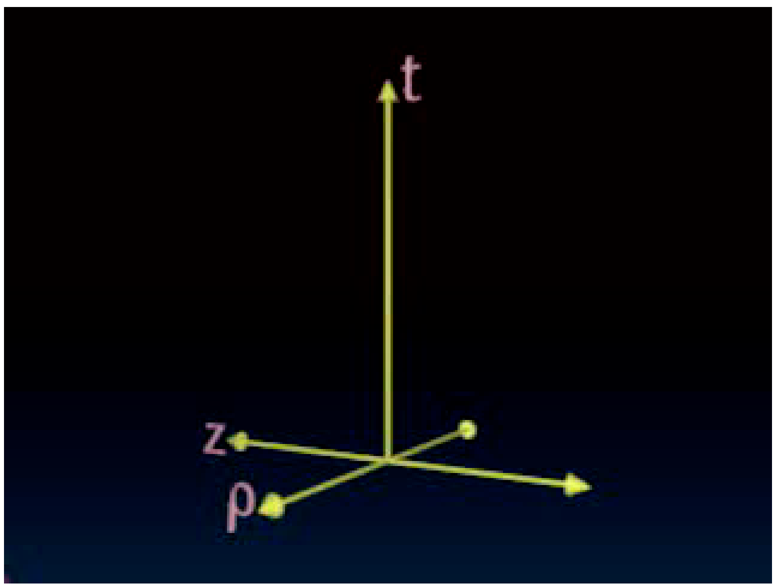
- Explicitly hard-code the divergences
- Do not let the punctures move

New View:

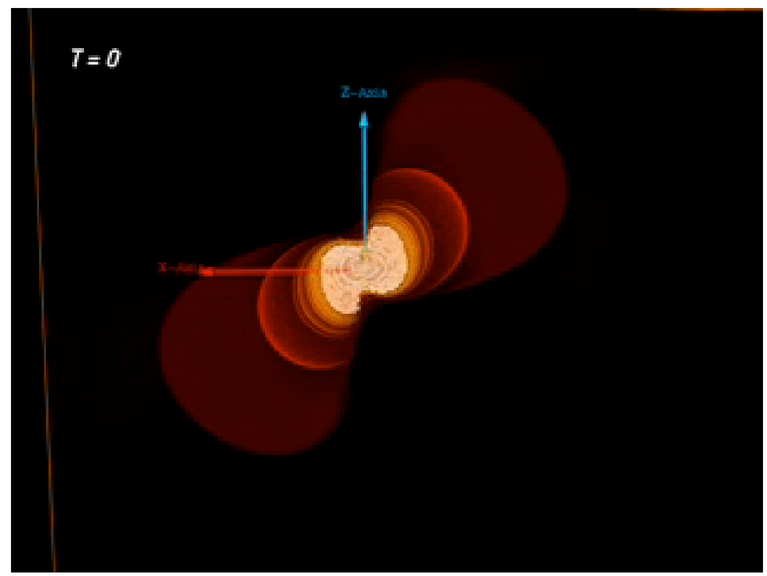
- Numerical smoothing will handle the singularities
- Let the punctures move

A Little Bit of History

1975-1980 - The Pioneer years
1994 - 2D Head-on Collisions
1995 - 3D Single Black Hole
1997 - Boosted BH
1999 - 3D Head-on Collisions
2000 - Grazing Collisions
2004 - Plunge Collisions

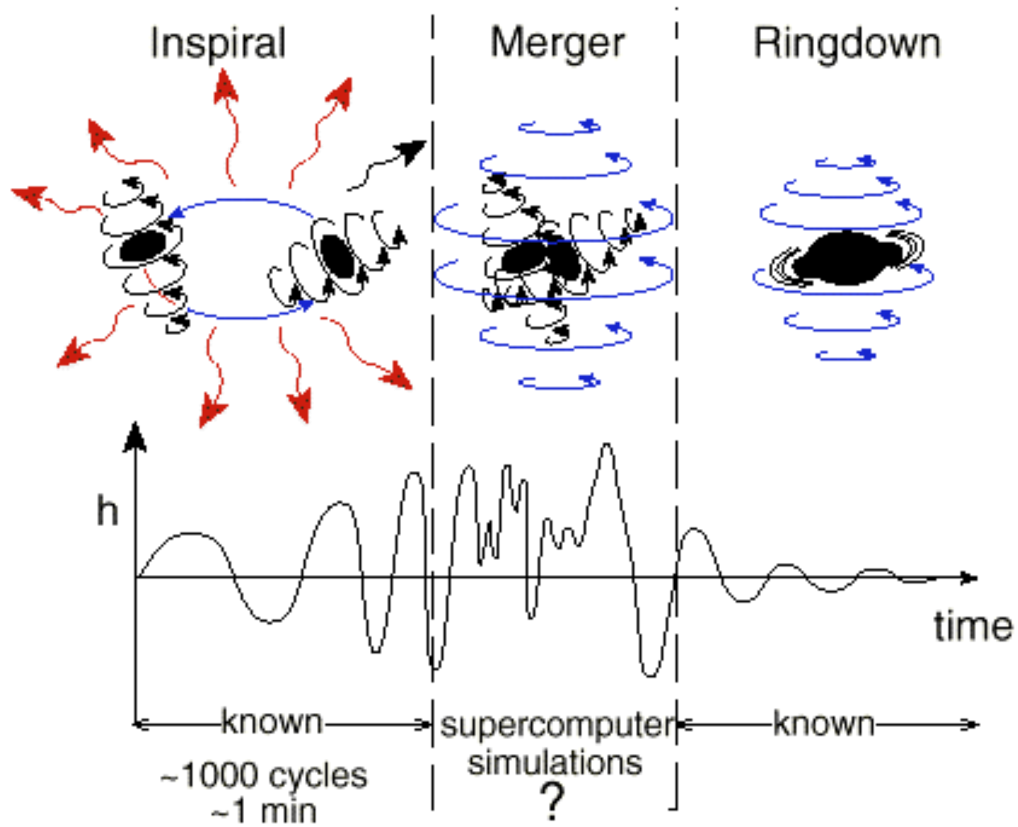


BBH Alliance

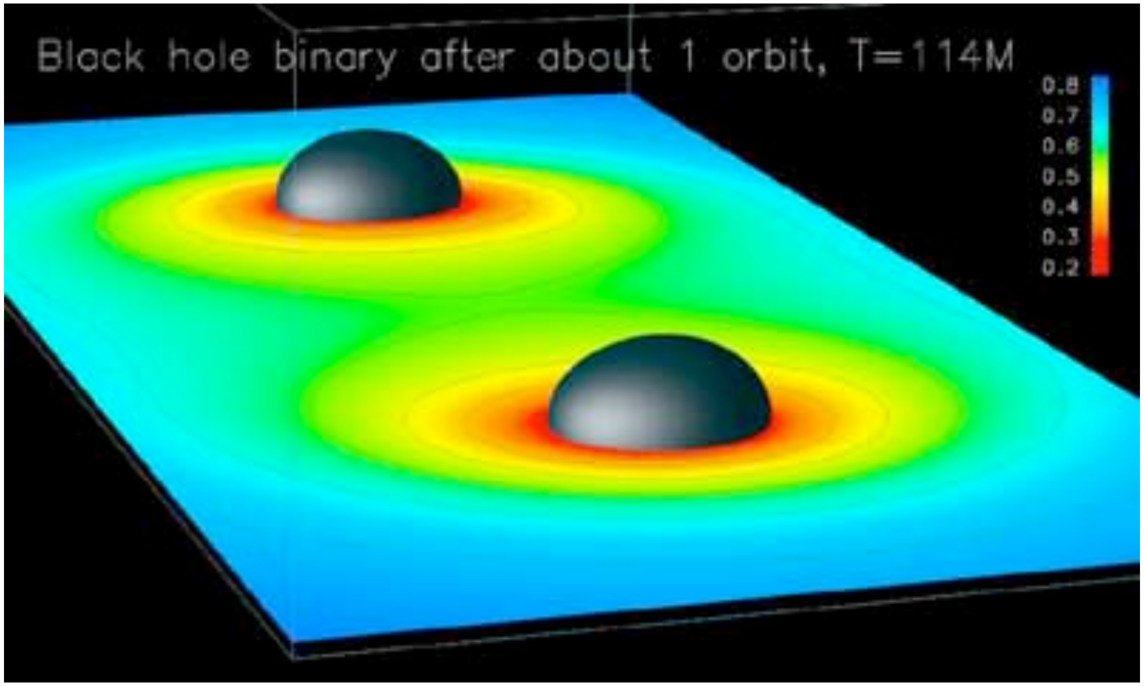


Albert Einstein Institute

The Ultimate Goal



First Orbit

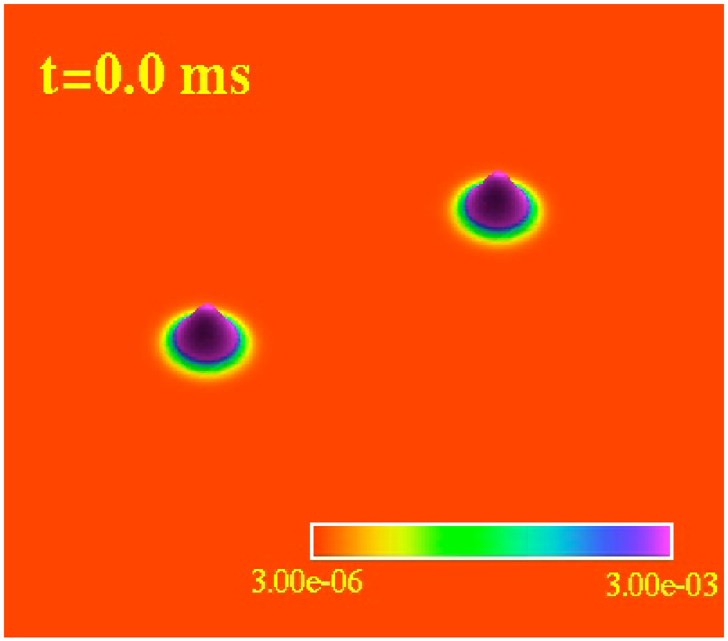


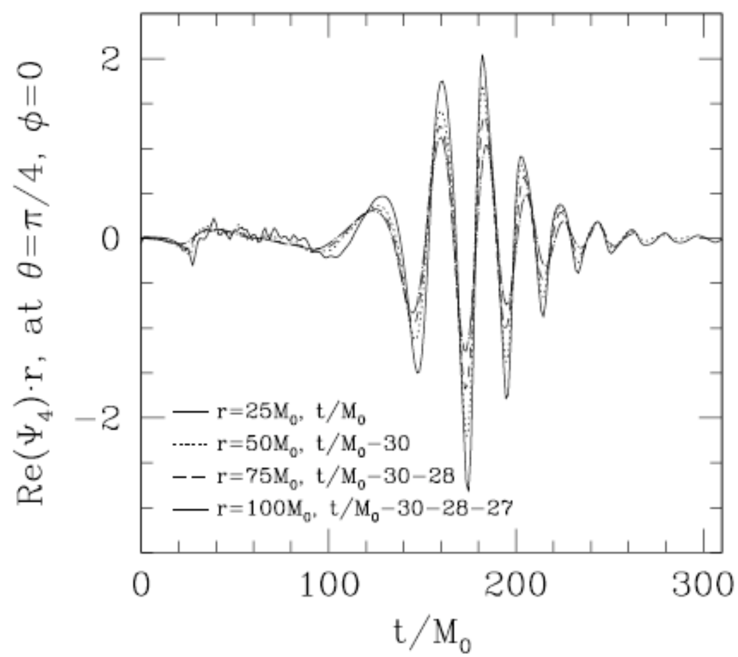
Bruegmann, Tichy, Jansen
PRL 92 (2004) 211101

- Co-rotating coordinate frame
- Evolution lasted 145 M
- Initial separation 6 M, Period ~ 114 M
- No waveforms were computed

First Orbits, Merger and Ring-down

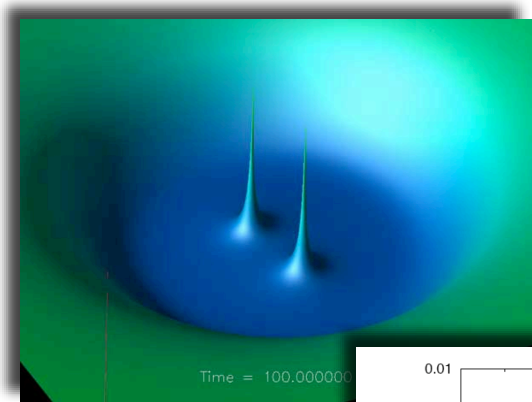
F. Pretorius (U. Alberta)
PRL 95 (2005) 121101

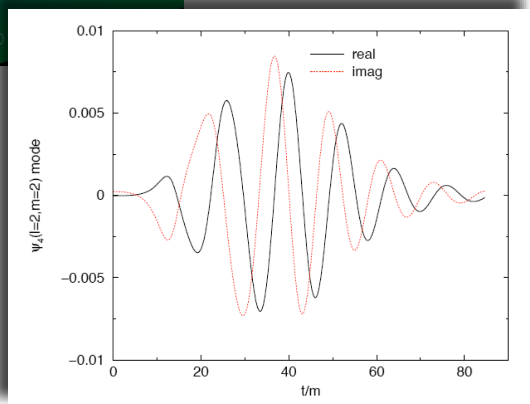


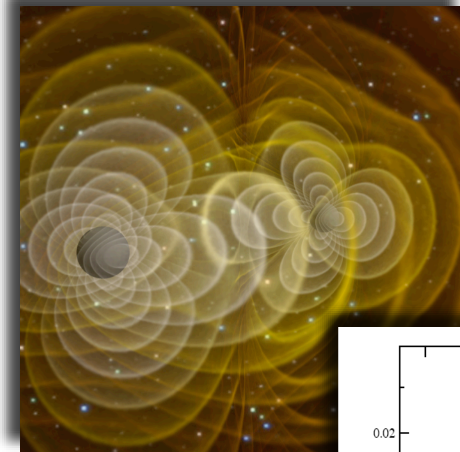


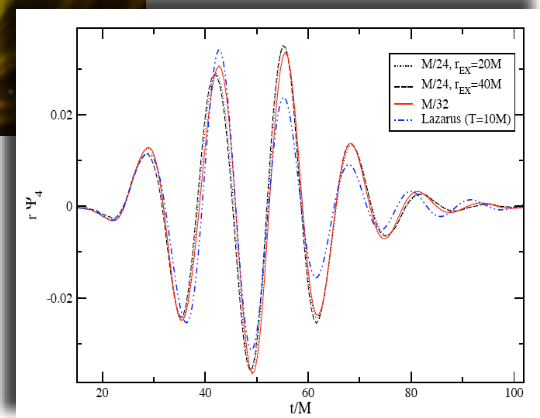
- Harmonic Formulation of Einstein Equations
- Initial data from unstable scalar field "stars"
- Initial separation ~ 16 M
- Eccentricity < 0.2
- Final BH spin a ~ 0.7

The Last Orbit:
The Moving Puncture Recipe









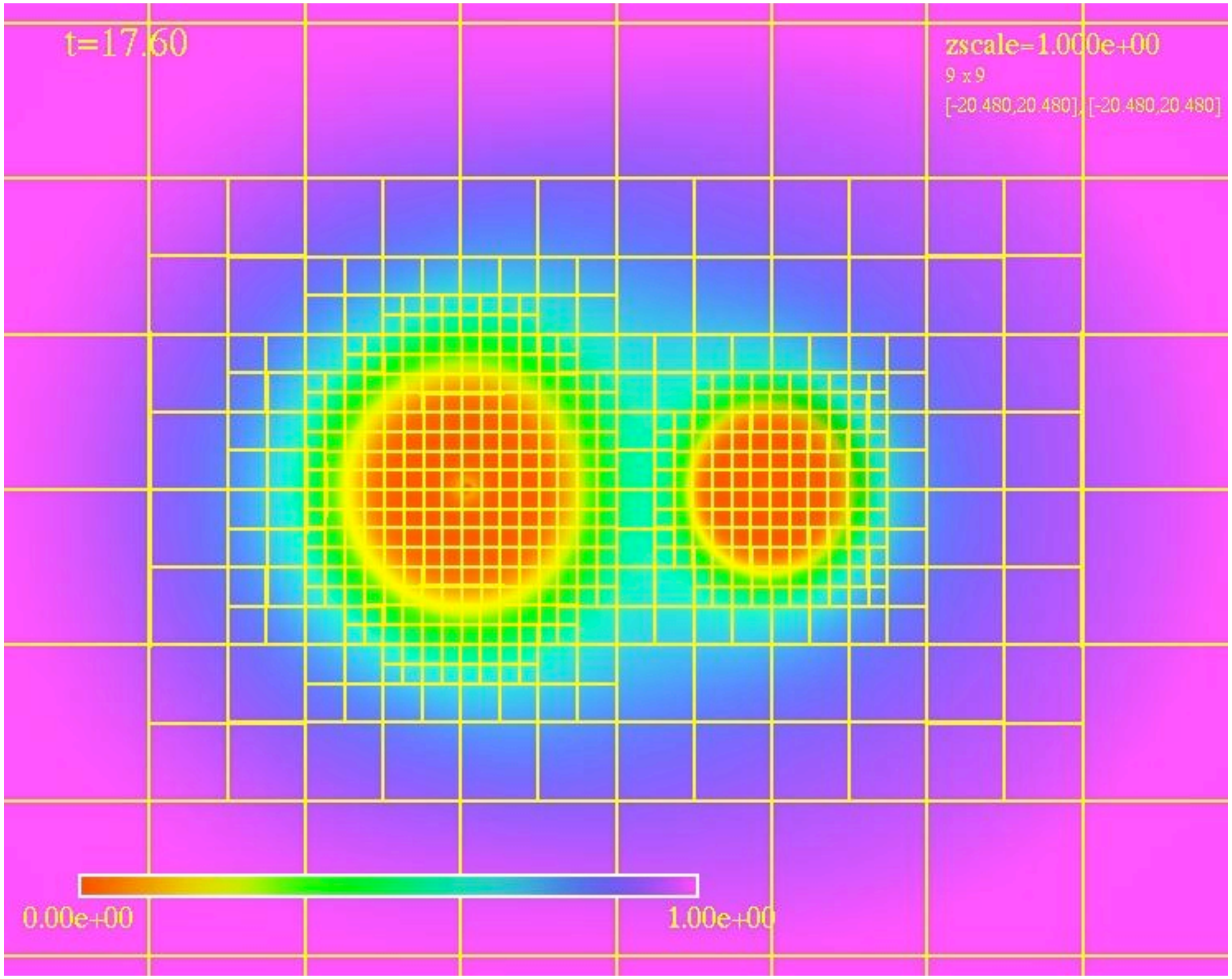
University of Texas at Brownsville
Campanelli, Louto, Zlochower
Phys.Rev.Lett. 96 (2006) 111101

NASA-GSFC
Baker, Centrella, Choi, Koppitz, van Meter
Phys.Rev.Lett. 96 (2006) 111102

Energy radiated ~ 3%, Ang. Mom. radiated ~ 15%, Final BH spin parameter ~ 0.7

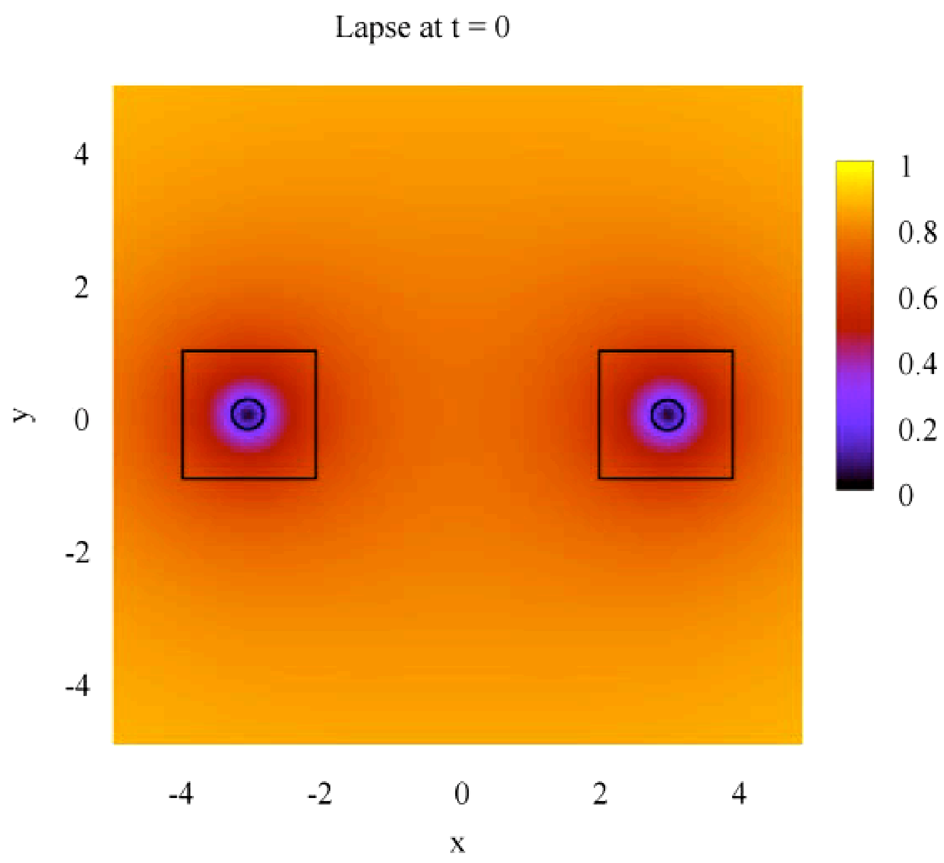
The Moving Puncture Recipe seems robust.
Penn State, AEI-LSU, Jena-FAU groups have successfully adopted.

Adaptive Mesh Refinement



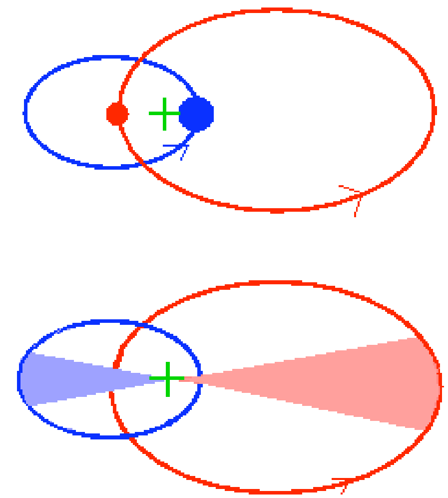
Dale Choi (NASA-GSFC)

Fixed Mesh Refinements



Herrman, Hinder, Shoemaker, Laguna

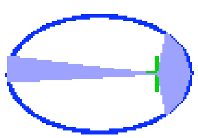
Gravitational Radiation Recoil: Black Hole Kicks



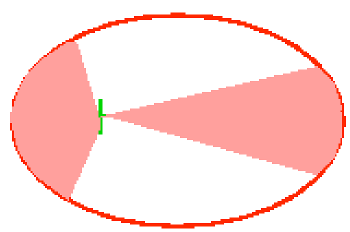
Kelper's 2nd Law: Each star sweeps out equal areas in equal times within its own orbit.

Thus, each stars travels at different speeds.

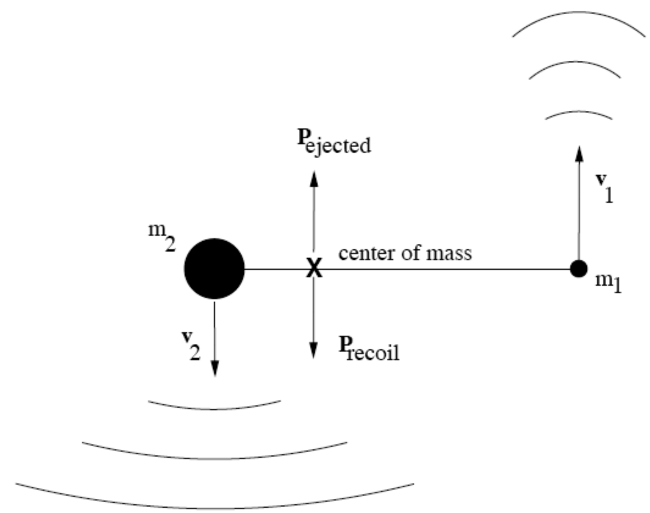
Slower star



Faster star

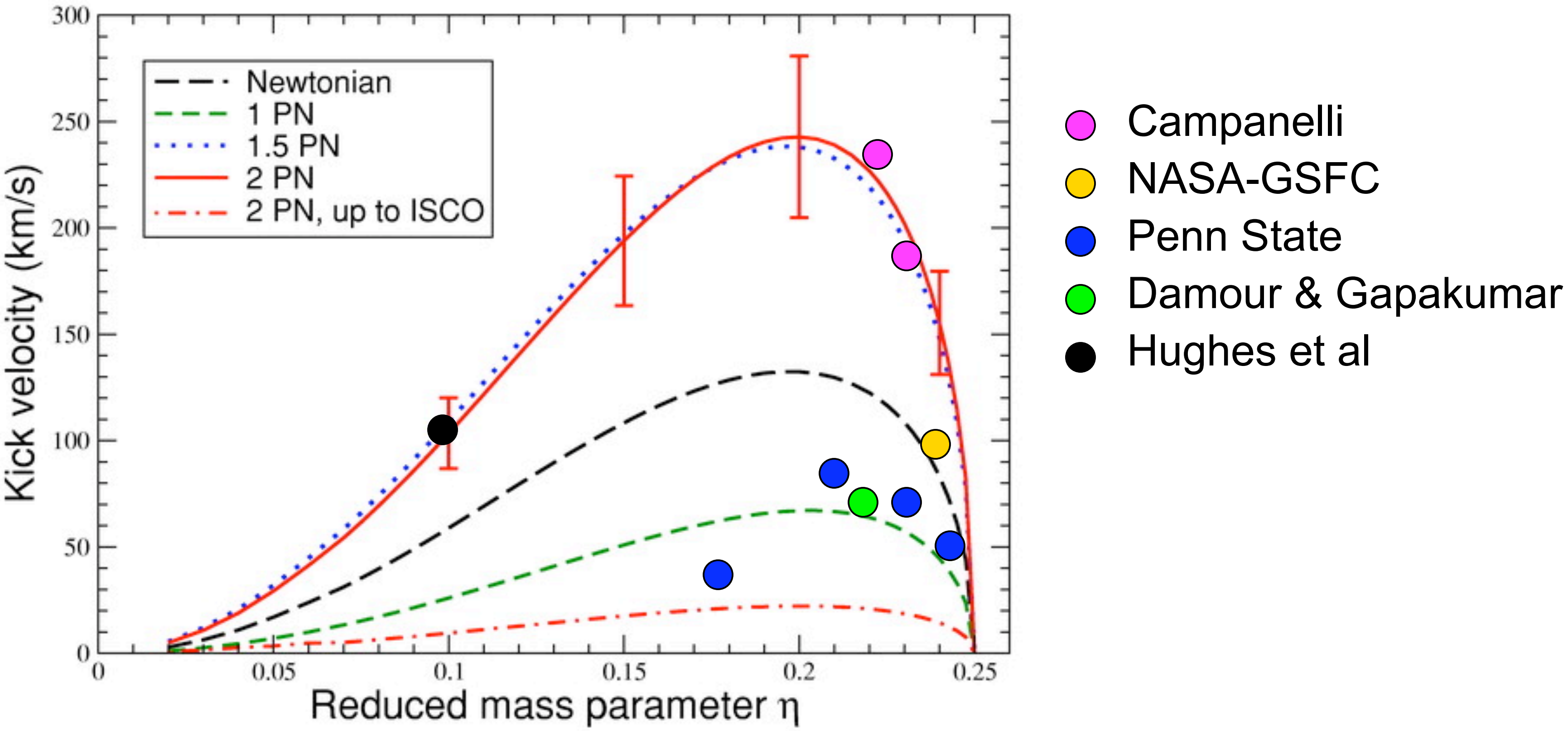


Scott A. Hughes et al.



Therefore, each star emits *"different"* amounts of energy and momentum

Kick Estimates

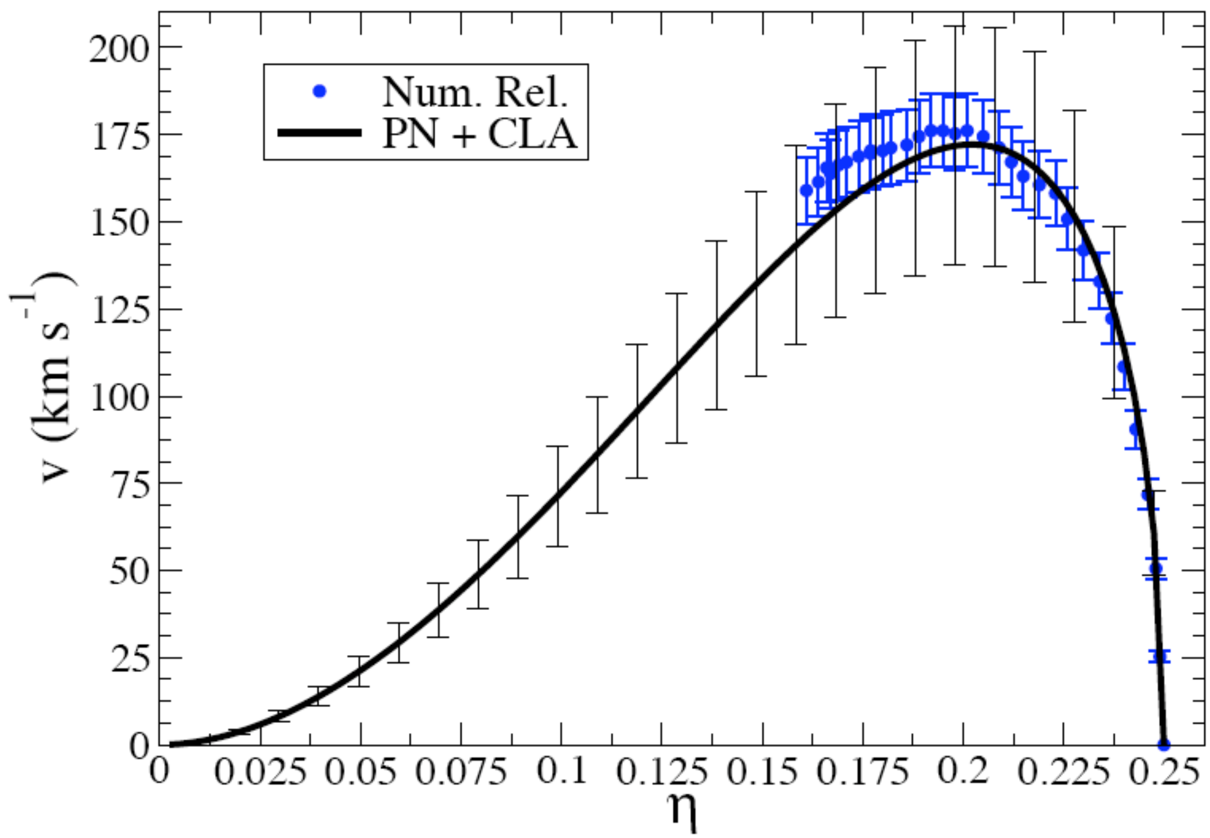


$$\eta = M_1 \, M_2 / (M_1 + M_2)^2$$

Blanchet, Qualiasah & Will (2005)

$$\frac{V}{c} = 0.043\eta^2 \sqrt{1 - 4\eta}\left(1 + \frac{\eta}{4}\right)$$

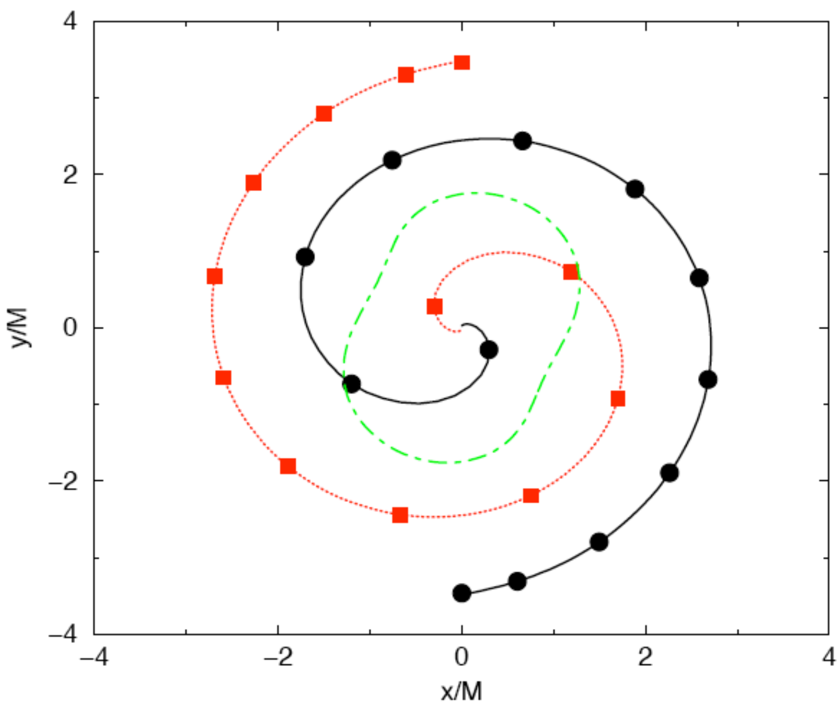
Kick Estimates



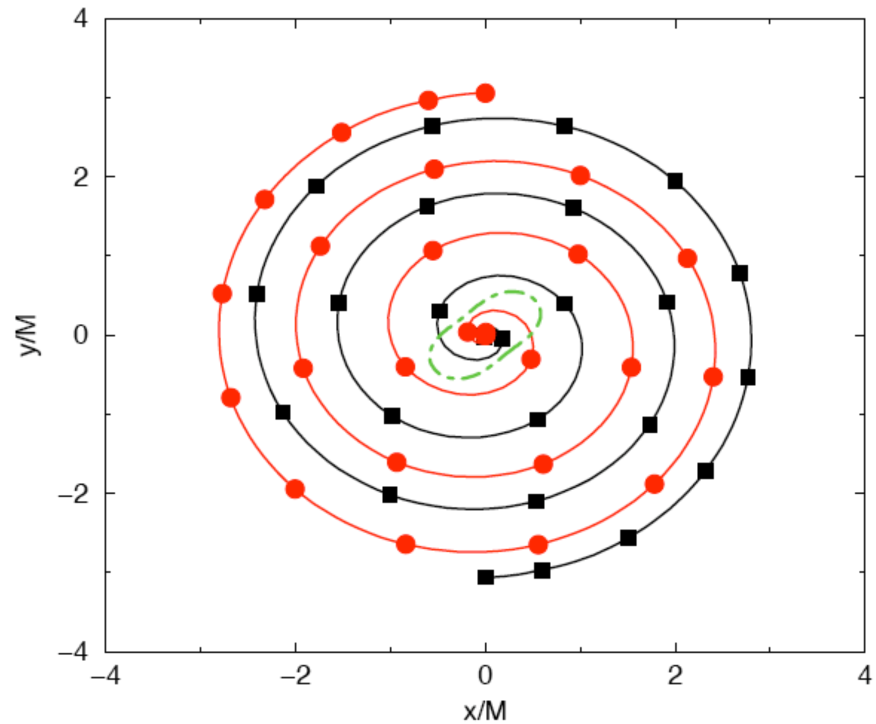
PN+CLA: Sopuerta, Yunes, Laguna astro-ph/0611110
Num Rel:Gonzalez et al gr-qc/0610154

BBH and Spins

Campanelli, Lousto, Zlochower gr-qc/0604012



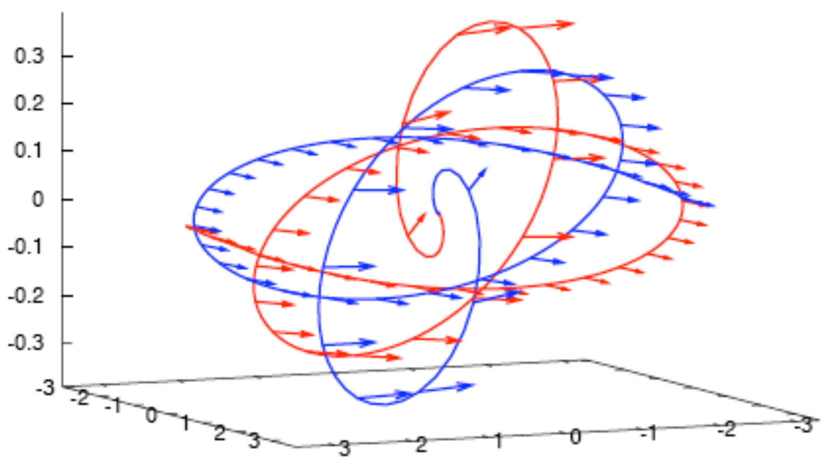
Spins Counter-aligned
Initial Spin= 0.75
Time to merge ~ 105.5 M
Energy Radiated ~ 2%
Ang. Mom. Radiated ~ 26%
Final Spin ~ 0.44

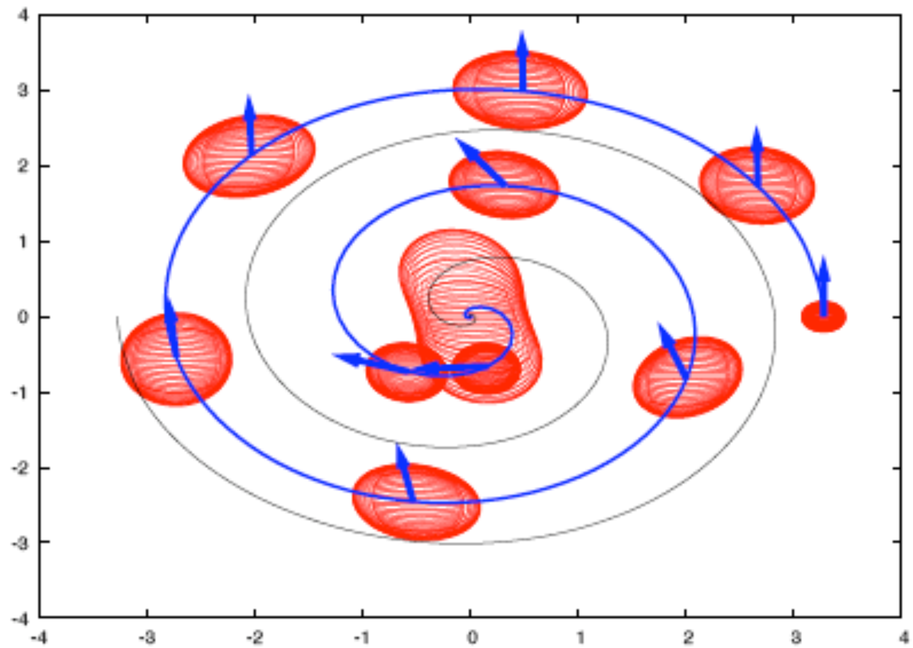


Spins Aligned
Initial Spin= 0.75
Time to merge ~ 224.5 M
Energy Radiated ~ 6%
Ang. Mom. Radiated ~ 33%
Final Spin ~ 0.9

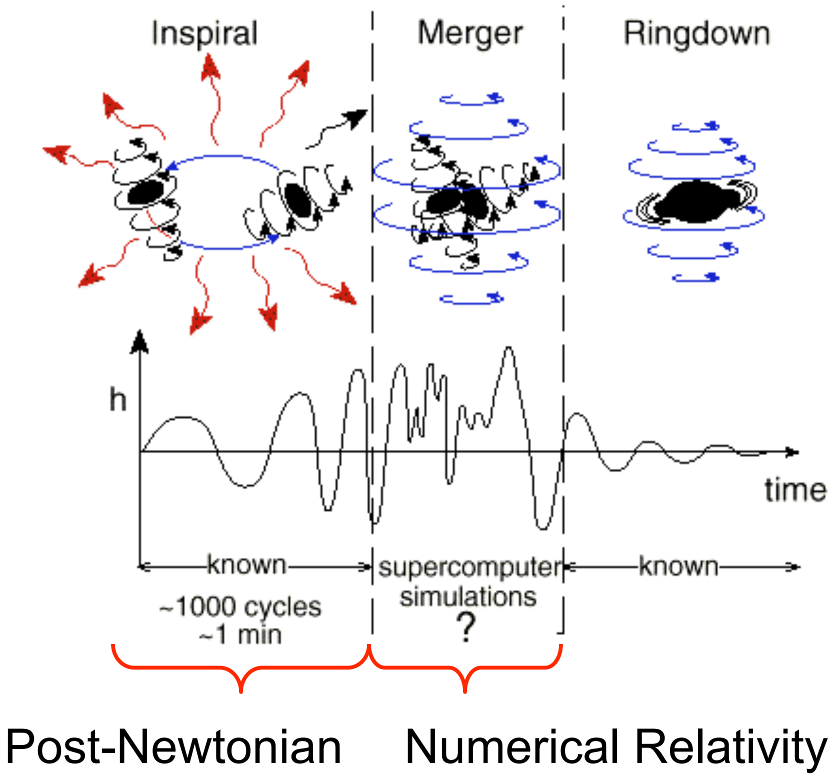
BBH and miss-aligned Spins

Campanelli et al gr-qc/0612076





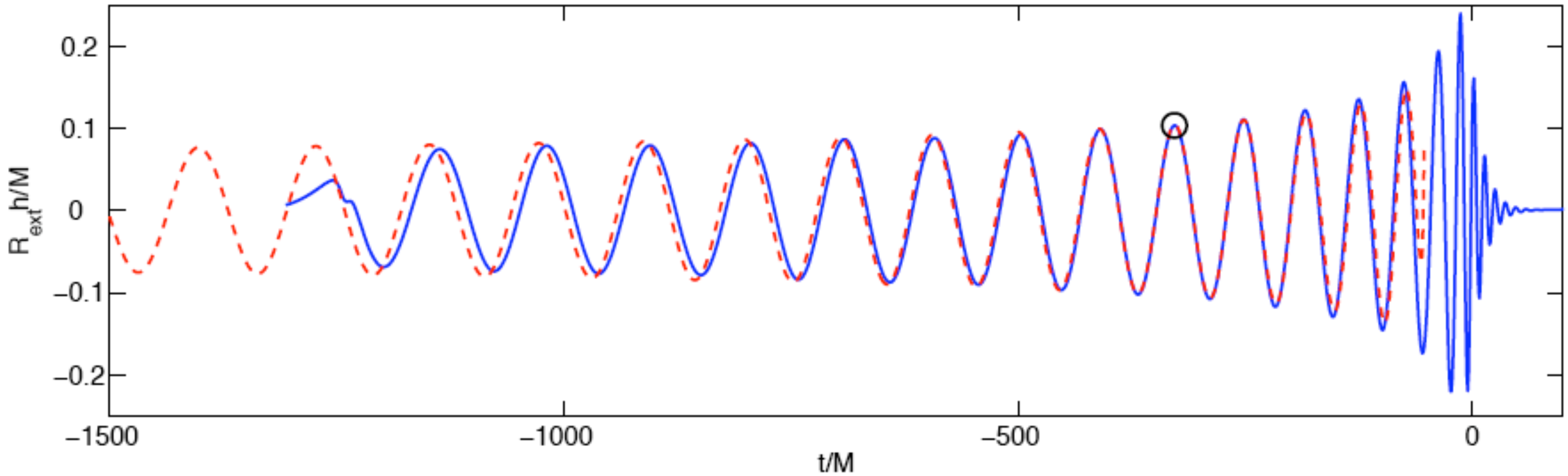
Bridging the Gap between Post-Newtonian and Numerical Relativity



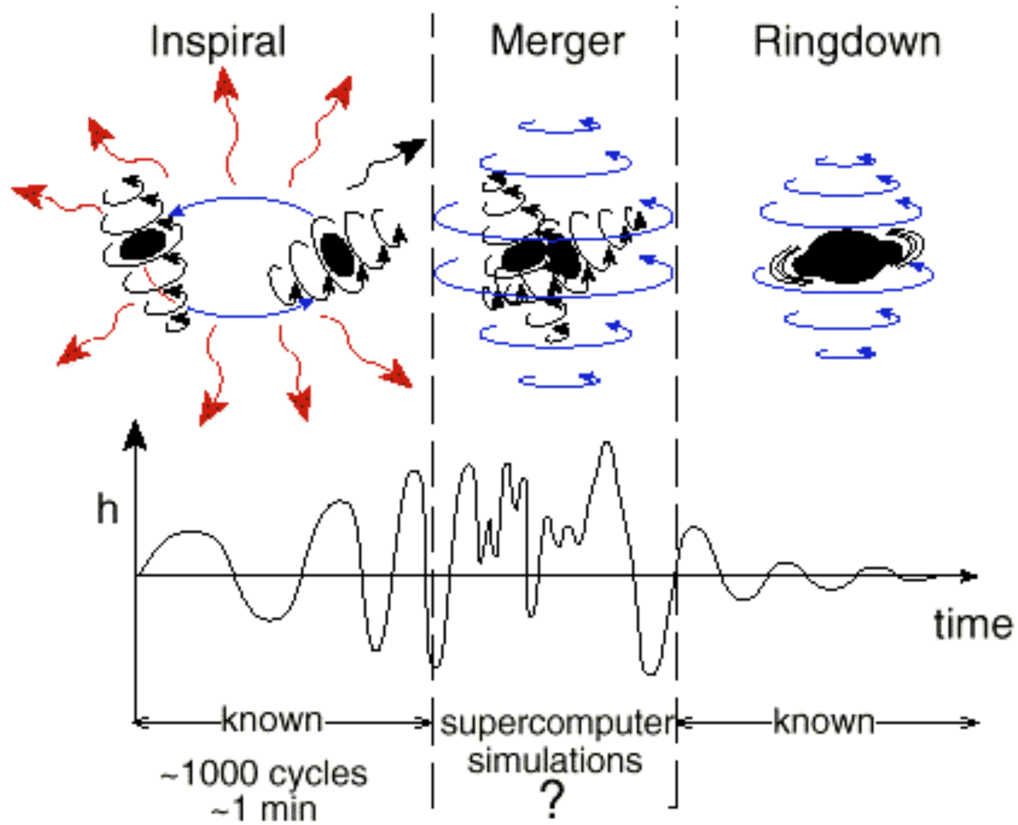
Is there a gap between the point in which PN approximations fails and NR simulations start?

Bridging the Gap between Post-Newtonian and Numerical Relativity

Baker et al gr-qc/0612117

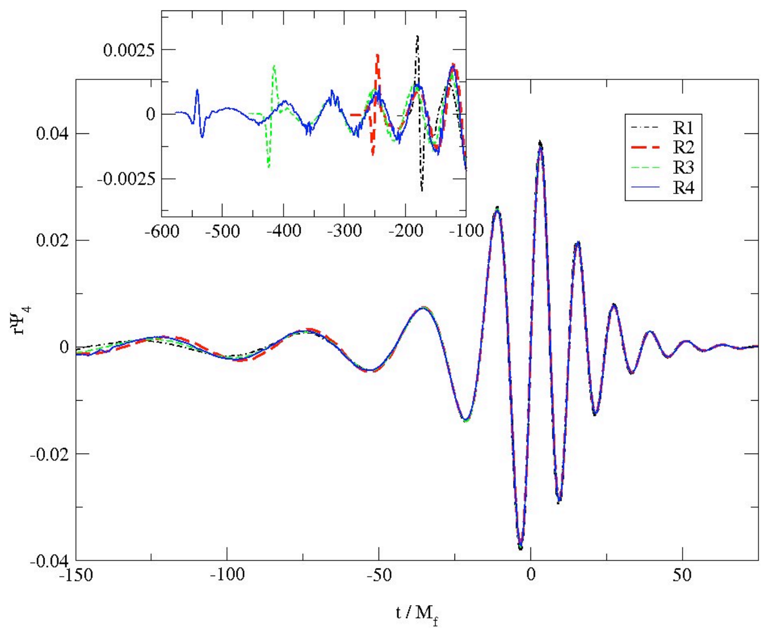


What happened to this picture?



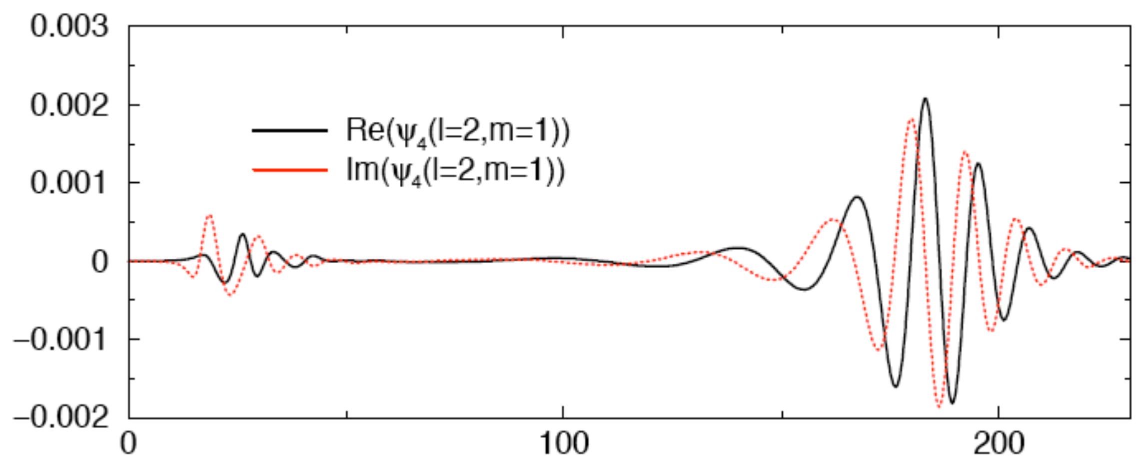
Where is the complexity in the gravitational waveforms?

So far, it seems not!

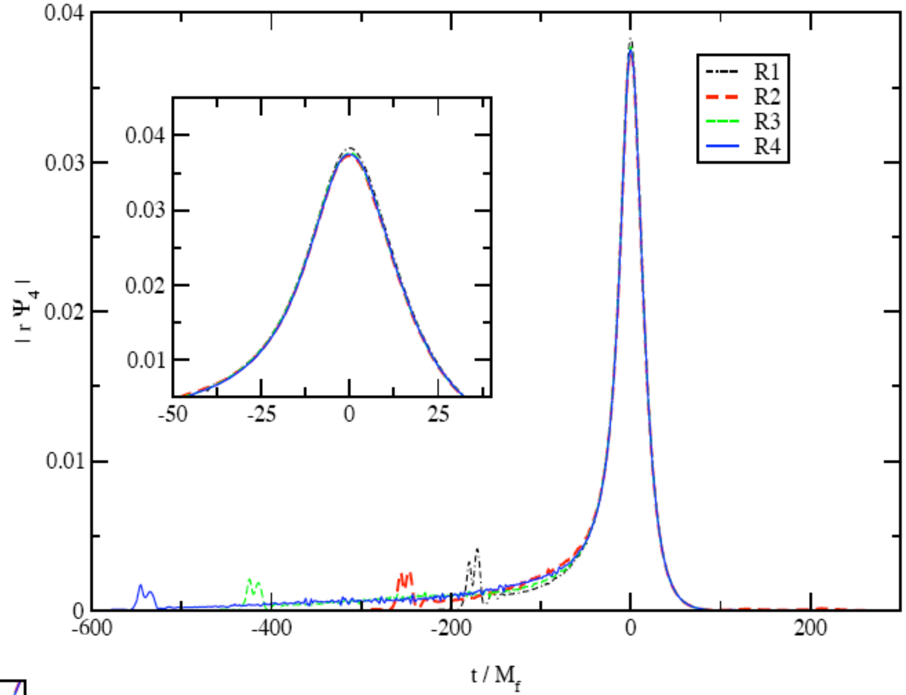


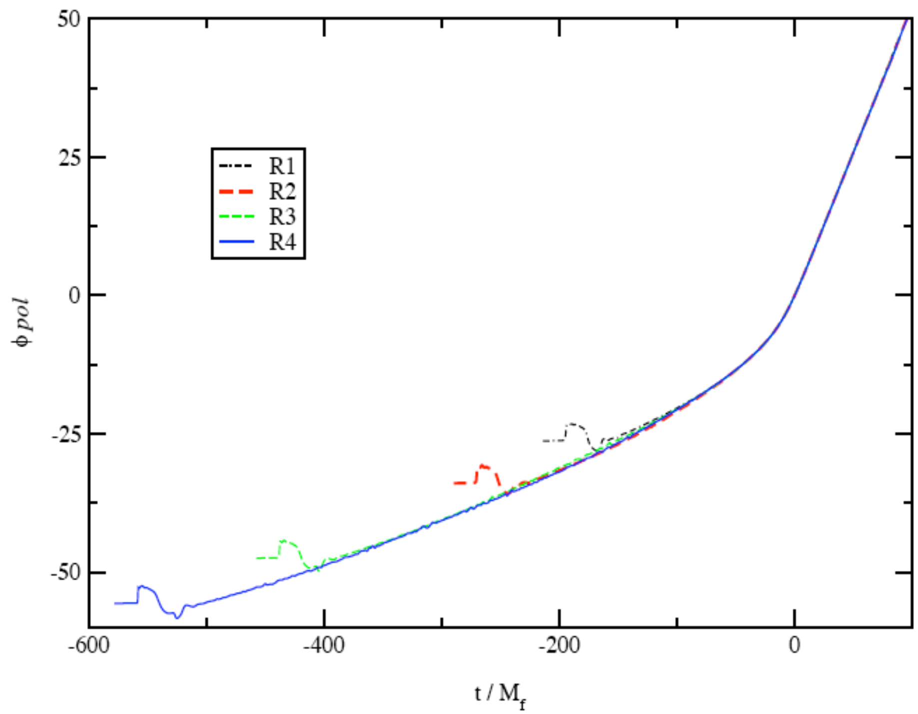
NASA-GSFC
Baker, Centrella, Choi, Koppitz, van Meter
Phys.Rev. D73 (2006) 104002

Campanelli et al gr-qc/0612076
BBH with spin flips!



Where is the complexity?





$$r\,\Psi_4 = A\,\exp\left(-i\,\phi\right)$$

What Next?

- Better Initial Data
- Better Computational Infrastructure
- Access to faster and larger hardware (peta-scale)
- Better bridge with Post-Newtonian results
- Connection with Data Analysis
- More on Spins and Un-equal Mass Binaries
- BH-NS Binaries
- Mathematical understanding of the moving puncture recipe

In Conclusion,

YES!