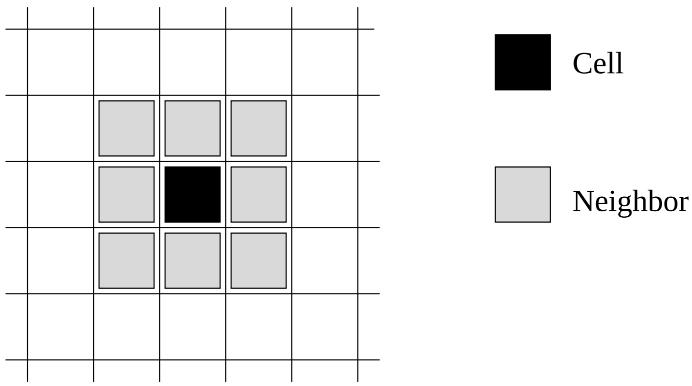
¶ 1. A cellular automaton is a discrete model consisting on a regular grid of cells, each one able to be in one of finitely many states. The collective states of the cells update at discrete times according to some a preset rules rules. Cellular automata theory began around the middle of the last century with work of von Neumann and Ulam, who where trying to designs self-reproducing machines. Today, Cellular automata theory is a very active of research in mathematics, computer science, physics, biology, and even sociology and linguistics.

¶ 2. The Game of Life is an example of cellular automaton devised by John H Conway, a mathematician at Princeton University.
The Game of Life takes place on a plane grid like graph paper. Each cell or unit square can be on any of two states: dead or alive. We represent a dead cell by a white square cell, and a live cell by a black square cell.
Each cell has 8 neighbors, those cells corresponding to the squares surrounding the given cell. Interaction with neighbors will determine whether a cell will die or be reborn.



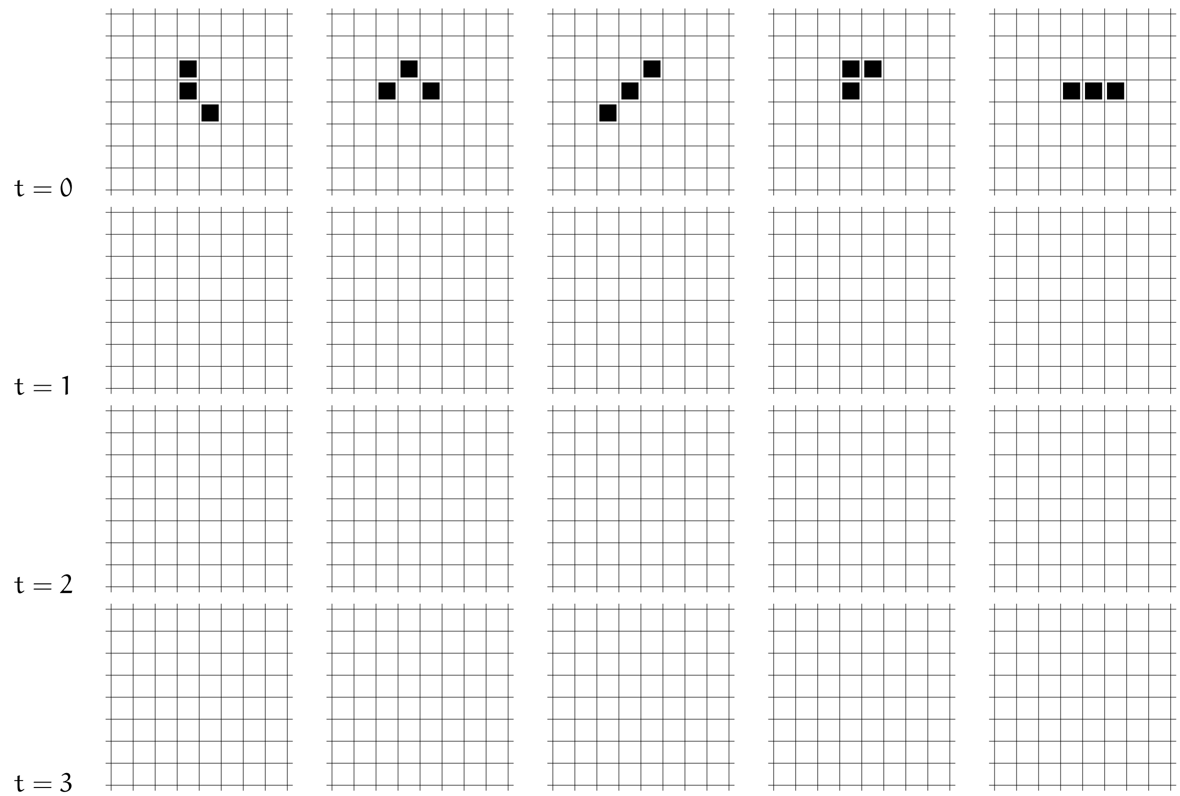
Configurations of cells will evolve at unit time intervals according to a set of preset rules. The rules for updating a configuration of cells are as follows:

1. (Survival) A live cell that has either two or three live neighbors remains a live cell.
2. (Deaths) A live cell that has 4 or more live neighbors dies from overcrowding. A live cell that has 1 or no neighbors dies from isolation.
3. (Births) A death cell that has exactly three live neighbors becomes a live cell.

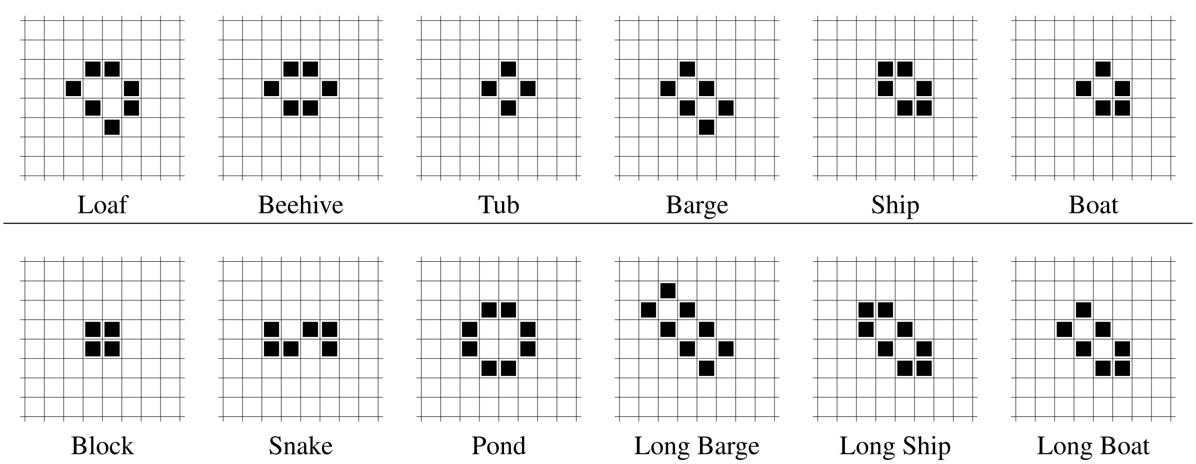
¶ 3. Tracking the evolution of a configuration of cells can be challenging at first because the updating rules must be applied collectively to all cell in the current configuration at once. Be methodical:

1. Locate all the live cells (black squares) that will die.
2. Locate all the live cells that will remain alive.
3. Locate all the death cells (white square) that will become live cells.

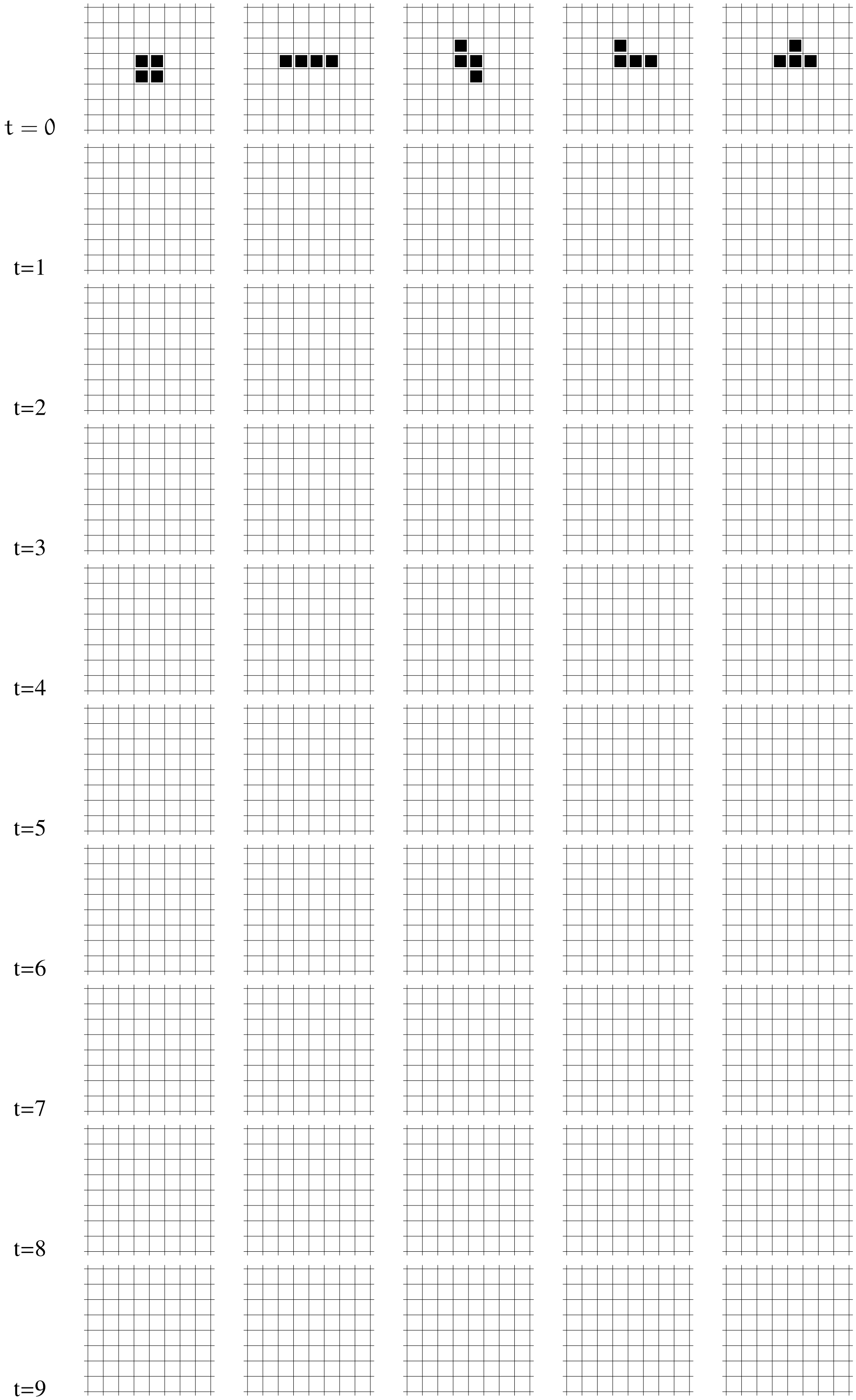
¶ **4.** Draw the first states of the three-cell configurations given in the top row.
You would have noticed that not all three-cell configurations behave the same way. Some die, some repeat, some are stable, and some evolve onto others that remain stable or that repeat.



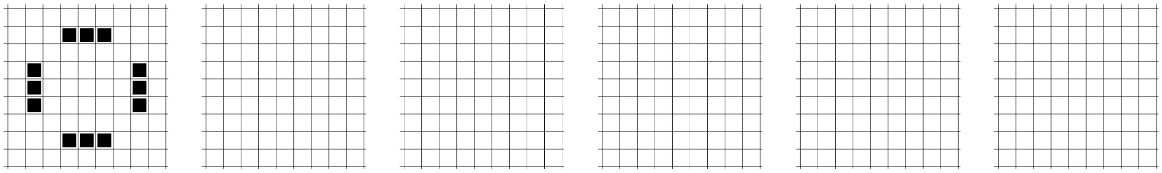
¶ **5.** You will have noticed that some configurations evolve into configurations that no longer change. These are called stable configurations. The most common ones are the following:



¶ **6.** Describe the evolution of the four-cell configurations depicted on the top row.

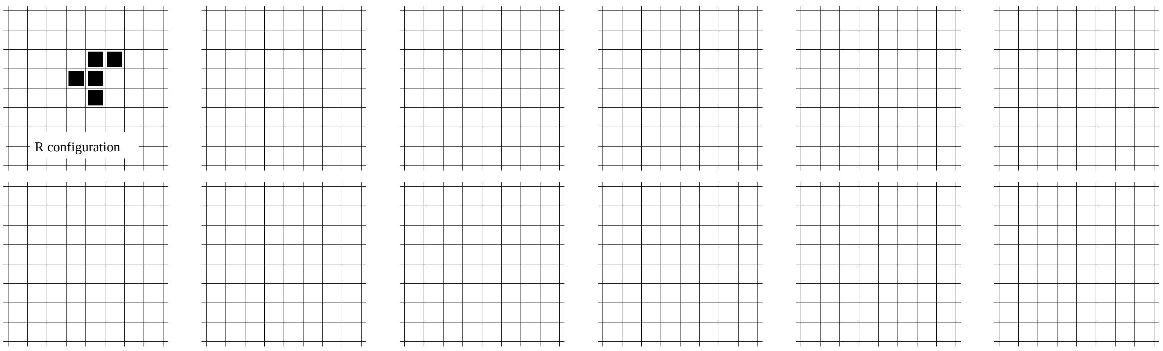


¶ **7.** This configuration is called **traffic light**. Can you study its evolution and explain its name?



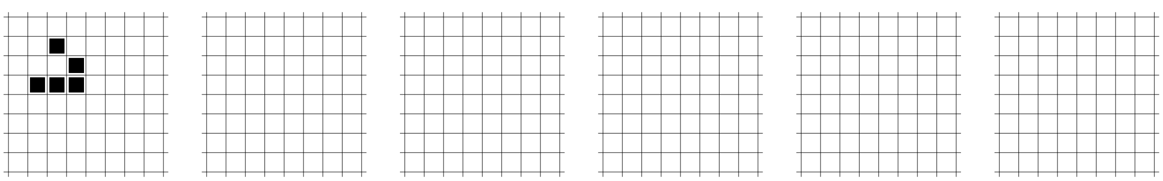
Traffic Light $t = 1$ $t = 2$ $t = 3$ $t = 4$ $t = 5$

¶ **8.** The behavior of five-cell configurations is more varied. There are 12 rook-wise connected five-cell configurations, up to symmetry. One of them is depicted below. Can you draw them all?



Of those five-cell rook-wise connected configuration, five will vanish before time $t = 5$, two will evolve into stable forms (a loaf), and two will become a "traffic lights" configuration. One other (the R-configuration above) will sticks around evolving into a multitude of shapes for quite sometime, too long to follow by hand.

¶ **9.** There is a remarkable five-cell configuration called the **glider**. After two steps, it has shifted and reflected itself in a diagonal line. After another two time steps, it ups itself and moved diagonally down to the right from its initial position.



Glider $t = 0$ $t = 1$ $t = 2$ $t = 3$ $t = 4$ $t = 5$

¶ **10.** Further analysis of the evolution of cell configurations in the Game of Life requires the use of a computer program. You may find several program on the World Wide Web, for example http://www.bitstorm.org/gameoflife

Literature

[1] Martin Gardner, *Penrose Tiles to Trapdoors Ciphers*, W. H. Freeman, 1989. (Chapters 20, 21, 22.)