¶ **1.** Five tables containing numbers are pictured below:

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

Table A

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

Table B

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

Table C

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

Table D

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

Table E

Answer the following two questions:

(a) Which tables contain the day of the month in which you were born?

(b) Which tables contain the month in which you were born? (January is 1, February is 2, and so on.)

Your celebrate your birth-date on:

¶ **2.** How does this trick work?

¶ **3.** Around 5,000 years ago, Egyptian mathematicians developed a method for multiplying two numbers using only addition and multiplication by 2. Here is how their method work for finding 37×13.

¶ 4. The inner workings of these tricks are based on Binary Notation and Binary Arithmetic. Our standard system for writing numbers is decimal notation (or base 10): a number is a sequence involving the ten digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, the position of any digit corresponding to a power of 10: the first digit from the right indicates the power $10^0 = 1$, the second digit from the right indicates the power $10^1 = 10$, and so on. For example, the number 297 is

$$297 = 2 \times 10^2 + 9 \times 10^1 + 7 \times 10^0$$

In binary notation (or base 2), we represent a number by a sequence of digits form $0, 1$, with their position corresponding to a power of 2: the first from the right indicates $2^0 = 1$, the second from the right indicates $2^1 = 2$, and so on. Thus, the number 1011 in binary notation is the number

$$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

or

$$1011 = 8 + 4 + 1 = 13,$$

in decimal notation. Thus a number can be represented in decimal notation or in binary notation: the digits may be different, but the number is the same.

¶ 5. To express the number 49 (written in decimal notation) in binary notation, we do the following: First find the largest power of 2 less that or equal to 51, which is $2^5 = 32$. Then subtract that fro 51 to get $51 - 32 = 19$. Again, find the largest power of 2 less than or equal to 19, which is $2^4 = 16$. Then subtract $19 - 16 = 3$, and find the largest power of 2 less than or equal to 3, which is $2^1 = 2$. Finally subtract $3 - 2 = 1$, which is the power $2^0 = 1$. Thus we have

$$\begin{aligned} 51 &= 2^5 + 2^4 + 2^1 + 2^0 \\ &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + +0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \end{aligned}$$

so 51 is 110011 in binary notation.

¶ 6. Find the binary notation for the following numbers written in decimal notation:

(a) 255

(b) 1027

¶ 7. Determine the decimal notation of the following numbers written in binary notation:

(a) 101010

(b) 110011

¶ **8.** Arithmetic in base 2 is performed as arithmetic in base 10 (our standard notation). For that we use the addition and multiplication tables for modulo 2 arithmetic, and remember to "carry":

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

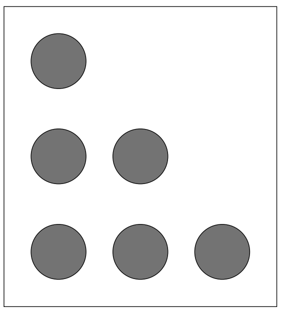
(a) Add the numbers written in binary notation:

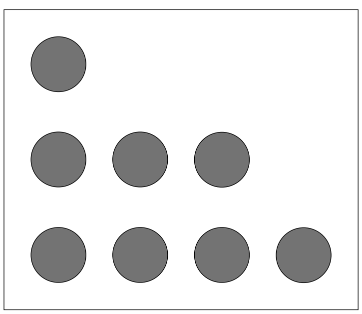
$$10010101 \qquad \text{and} \qquad 1000101.$$

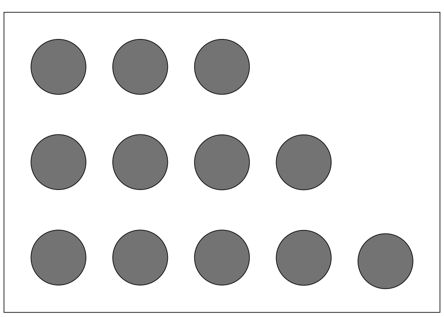
(b) Multiply the numbers written in binary notation:

$$1011 \qquad \text{and} \qquad 1101011.$$

¶ 9. Binary notation has many uses: form theoretical applications in mathematics and computer science, to more practical ones in all areas of engineering and technology. It also gives rise to many problems in recreative mathematics, like the birthdate problem at the beginning of this handout.
Here is a more sophisticated one to game theory. The game of Nim is played by tow persons. Any number of counters is arranged on a table into several heaps. The two players alternate in removing one or more counters from any single heap. Whoever takes the last counter (or counters) wins the game. This is a game for which one of the player has a winning strategy. The strategy is rather simple to explain, but not easy to figure out.







Literature

[1] W. W. Rouse Ball, *Mathematical Recreations and Essays*, McMillan, New York, 1962.

[2] Martin Gardner, *New Mathematical Diversions*, he Mathematical Association of America, 1995.