

## Math 655. Homework 3. Due 2/26/03

**Problem 1** Let  $f$  be an analytic function on a connected open set  $U \subset \mathbf{C}$ .

- (1) Show that if  $f$  is real valued, then  $f$  is constant on  $U$ .
- (2) Show that if  $f$  has constant absolute value, then  $f$  is constant on  $U$ .

**Problem 2** Let  $f$  be analytic on  $\mathbf{C}$  and real valued on  $|z| = 1$ . Show that  $f$  is constant.

**Problem 3** Let

$$f(z) = \int_{[1,z]} \frac{1}{w} dw$$

where  $[1, z]$  is the line segment from 1 to  $z$  in  $\mathbf{C}$ . Show that  $f$  is a well defined analytic function on  $\mathbf{C} \setminus \{z = x + iy \mid x \leq 0\}$ , and compute its power series expansion centered at the point  $z_0 = 1$ .

**Problem 4** Show that if  $P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$  is a polynomial of degree  $n \geq 1$ , then  $|P(z)| \rightarrow \infty$  as  $|z| \rightarrow \infty$ . In fact, show that if  $|z| \geq \max\{1, 2n|a_{n-1}|, \dots, 2n|a_0|\}$ , then  $|P(z)| \geq |z|^n/2$ .

**Problem 5** Let  $f$  be an entire function such that

$$|f(z)| \leq A|z|^k$$

for all  $z \in \mathbf{C}$ , for some constant  $A$  and integer  $k$ . Show that  $f$  is a polynomial of degree  $\max\{0, k\}$ .