Math 655. Homework 1. Due 2/5/03

Problem 1 Prove that

$$\left|\frac{a-b}{1-\overline{a}b}\right| < 1$$

if $|a| < 1$ and $|b| < 1$.

Prove also that

$$\left|\frac{a-b}{1-\overline{a}b}\right| = 1$$

if either $|a| = 1$ or $|b| = 1$. What exception must be made if $|a| = |b| = 1$?

Problem 2 Show that the functions $f(z)$ and $\overline{f(\overline{z})}$ are simultaneously analytic.

Problem 3 Evaluate the integral

$$\int_\gamma \overline{z}\, dz,$$

where γ is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$.

Problem 4 If the power series $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence R, show that the differentiated series $\sum_{n=1}^{\infty} n a_n z^{n-1}$ also has radius of convergence R.

Problem 5 A function f defined on an open set U has a primitive on U if there is a function $F : U \to \mathbf{C}$ such that $F' = f$ on U. Show that the function $f(z) = 1/z$ has no primitive on $0 < |z| < 1$.