HOMEWORK 8. DUE 10/17/02

Problem 2.1.1. Given a set X and subsets $A_1, A_2, A_3, \cdots$, let

$$A_+ = \limsup A_n = \bigcap_{k=1}^{\infty} \bigcup_{n \geq k} A_n$$

and

$$A_- = \liminf A_n = \bigcup_{k=1}^{\infty} \bigcap_{n \geq k} A_n.$$

Let f_+, f_- and f_n be the characteristic functions of the set A_+, A_- and A_n, respectively. Prove that $f_+ = \limsup f_n$ and $f_- = \liminf f_n$.

Problem 2.1.3. (a) Let $f_n : X \to \mathbf{R}$ be a sequence of measurable functions. Show that the set $\left\{x \in X \;\middle|\; \text{the sequence} f_n(x) \text{ converges}\right\}$ is measurable.

(b) Deduce that the set of points $\omega \in I$ for which the randomized series

$$\sum_{k=1}^{\infty} \frac{R_k(\omega)}{k}$$

converges, is a measurable subset of I.

Problem 2.1.8. Let $f : \mathbf{R} \to \mathbf{R}$ be monotone increasing. Show that f is measurable.