MATH 650. HOMEWORK 7. DUE 10/3/02

Problem 1. Let $(X, \mathcal{F}, \mu)$ be a probability space. Let $A_1, A_2, \ldots$ be a sequence of subsets of X belonging to $\mathcal{F}$.

(a) Show that if $A_1 \supset A_2 \supset A_3 \supset \cdots$, then

$$\mu(\bigcap_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n).$$

(b) Show that if $A_1 \subset A_2 \subset A_3 \subset \cdots$, then

$$\mu(\bigcup_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n).$$

Problem 2. Let X be a probability space and A_n measurable sets. Show that the probability of $\liminf_n A_n^c$ is 0 if and only if the probability of $\limsup_n A_n$ is 1.

Problem 3. Let $(X, \mathcal{F}, \mu)$ be a probability space, and let $A_1, A_2, \ldots$ be in $\mathcal{F}$. Show that

$$\mu(\liminf_n A_n) \leq \liminf_n \mu(A_n) \leq \limsup_n \mu(A_n) \leq \mu(\limsup_n A_n).$$

Problem 4. Let $(X, \mathcal{F}, \mu)$ be a probability space. Show that if $A_1, A_2, \cdots, A_n$ are independent sets from $\mathcal{F}$, then the sets $A_1^c, A_2, \cdots, A_n$ are also independent.