MATH 650. HOMEWORK 6. DUE 9/26/02

Problem 1. Let A be a measurable subset of $\mathbf{R}$. The density of A is well defined if the limit

$$D(A) = \lim_{T\to\infty} \frac{\mu_L(A \cap [-T, T])}{2T}$$

exists.

(1) Find a measurable set whose density is well defined.
(2) Show that if A and B have well-defined density and are disjoint, then $A \cup B$ has a well-defined density and

$$D(A \cap B) = D(A) + D(B)$$

(3) Show that there exists sets A and A_n, $n = 1, 2, \cdots$, with well-defined densities such that $A = \bigcup_{n=1}^{\infty} A_n$ (disjoint union) but

$$D(A) \neq \sum_{n=1}^{\infty} D(A_n).$$

Problem 2. Let X be an uncountable set. Let $\mathcal{R}$ be the collection of all finite subsets of X. Given $A \in \mathcal{R}$ let $\mu(A)$ be the number of elements in A.

(1) Show that $\mathcal{R}$ is a ring and that μ is a measure on $\mathcal{R}$.
(2) Identify μ^*
(3) What are $\mathcal{M}$ and $\mathcal{M}_F$?
(4) Is every subset of X measurable?
(5) $(X, \mathcal{M}, \mu^*)$ is not σ-finite.

Problem 3. Show that given any $\delta > 0$ there exists an open dense subset U of $\mathbf{R}$ with Lebesgue measure $\mu_L(U) < \delta$.