

Math 592D. Homework 2. Due: 3/1/05

1. Under a continuous model, the population $N(t)$ follows a logistic model.

$$\frac{dN}{dt} = rN(t) \left(1 - \frac{N(t)}{K} \right).$$

where $r > 0$ and $K > 0$ are some parameters. We have seen that the steady state for a population following this model is $N^* = K$.

This population is subject to harvesting at a rate $h(t)$, so that the change in population with time is now

$$\frac{dN}{dt} = rN(t) \left(1 - \frac{N(t)}{K} \right) - h(t). \quad (1)$$

- Assume that $h(t) = h$ is constant. Determine the the steady states of the model described by Equation (1).
- Prove that there is a maximum sustainable yield h_M with the property that any larger harvest rate will lead to the depletion of the population.

2. Fishery statistics normally include information regarding the fishing effort. This is measured in units pertinent for the fishery in question: the number of ships-day per unit time; the number of nets, traps.

The ratio of catch divided by effort is a rough indication of the current stock level of the fish population. Assume that the catch-per-unit-effort is proportional to stock level, in equations

$$h = cEx$$

where E denotes the effort and c is a constant called the catchability coefficient. The product of catchability and effort, cE , is called the fishing mortality. Upon substitution into the basic harvesting model we obtain

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - cEN \quad (2)$$

- Find the steady states of equation (2) and study their stability.
 - Sketch the bifurcation diagram for this harvesting model, that is, draw the location of the steady state N^* as a function of the fishing mortality cE . This is a curve in the (cE, N^*) -plane which has several branches (stable-unstable). Where these branches collide (bifurcation points) there is a qualitative change in the behavior of the system. Do they have any biological meaning?
 - Find the maximum sustainable yield and the corresponding optimal effort level.
3. So far we have been assuming that fish populations grow according to the logistic model. However, some populations possess a threshold to grow. One such model is

$$\frac{dN}{dt} = rN \left(\frac{N}{K_0} - 1 \right) \left(1 - \frac{N}{K} \right) \quad (3)$$

- Determine the steady states for a population whose growth model satisfies the equation above, and study their stability.
- Suppose that such population is subjected to harvesting $h(t) = cEN(t)$. Draw the bifurcation diagram as in Problem 2.

Readings

You can download the following articles from www.jstor.org (you must be connecting from CSUN). They provide some bio-economic background to the problems in this homework set.

1. Bob Holmes (1994) *Biologists sort the lessons of fisheries collapse*, *Science* **264**, pp. 1252–1253.
2. Marcia Barinaga (1995) *New study provides some good news for fisheries*, *Science* **269**, p. 1043.
3. R. Myers et al. (1995) *Population dynamics of exploited fish stocks at low population levels*, *Science* **269**, pp. 1106–1108.