# Math 592D. Homework 1. Due: 2/10/05

**1.** The US Constitution requires that a census of the US population be taken every 10 years, starting 1790. Here is the US Census data that has been collected so far:

| Year | Population | Year | Population |
|---|---|---|---|
| 1790 | 3,929,214 | 1900 | 75,994,575 |
| 1800 | 5,308,483 | 1910 | 91,972,266 |
| 1810 | 7,239,881 | 1920 | 105,710,620 |
| 1820 | 9,638,453 | 1930 | 122,775,046 |
| 1830 | 12,866,020 | 1940 | 131,669,275 |
| 1840 | 17,069,453 | 1950 | 151,325,798 |
| 1850 | 23,191,873 | 1960 | 179,323,175 |
| 1860 | 31,433,321 | 1970 | 203,302,031 |
| 1870 | 39,818,449 | 1980 | 226,545,805 |
| 1880 | 50,155,583 | 1990 | 248,709,873 |
| 1890 | 62,947,714 | 2000 | 281,421,906 |

(a) Find an exponential growth model for the US population between 1790 and 1930, and use that to predict the population at later years.

(b) Use the method explained in class to get a logistic model for the US population (that is, find the parameters that give the best linear fit).

**2.** The attached article by Stephen Jay Gould makes the observation that most species of snails consist of animals whose shells curl almost exclusively one way.

One mathematical model discussed in the book by C. H. Taubes (*Modelling Differential Equations in Biology*, Prentice Hall, 2001) gives a simple mathematical model to predict the bias of either the left or right-handed forms for a given species.

1. Assume that the probability of a snail of type A (left or right) breeding with a snail of type B is proportional to the product of the number of type $A$ times the number of type $B$.

2. Assume that two left snails always produce a left snail and two right snails produce a right snail, and that a left-right coupling produces left and right offspring with equal probability.

These assumptions are open to discussion.

(a) Show that the first assumption implies that given a choice of mates a snail is twice as likely to choose one of its kind (i.e., left or right) that of the opposite kind.

You can also see from this assumptions that when there are more left than right handed snails, there will be more than the proportionate number of left offspring, and viceversa. For example, if the ratio left to right is $2:1$, then the ratio of left babies to right ones will be about $5:2$.

Taubes presents the following model that qualitatively exhibits the behavior described by the assumptions above. Let p(t) be the percentage (or probability) of left-handed conch shells at time $t$. Then

$$\frac{dp}{dt} = \alpha p(1-p)(p-1/2),$$

where $\alpha$ is some positive constant.

(b) Determine the equilibrium solutions of this equation. Which are stable? Which are unstable? What is the behavior of other solutions?