Math 550. Homework 4. Due 10/01/2003

Problem 1 Given a 1-form ω on an open set U, prove that the following are equivalent (i) $d\omega = 0$; (ii) $\int_{\partial R} \omega = 0$ for all closed rectangles R contained in U; (iii) every point in U has a neighborhood such that $\int_{\partial R} \omega = 0$ for all closed rectangles contained in the neighborhood. Is the same true if closed rectangles are replaced by disks?

Problem 2 Let $H : U \to \mathbf{R}^2$ be a smooth function. Show that $dH^* = H^* d$.

Problem 3 Let R be a region in the plane between two concentric circles γ_1 and γ_2 of radius $r_1 < r_2$. Prove that if U is an open set containing R and ω is a 1-form on U, then

$$\int_R d\omega = \int_{\partial R} \omega,$$

where $\partial R = \gamma_2 - \gamma_1$.

Problem 4 Prove that the relation of being homotopic relative to endpoints, or homotopic as closed paths, is an equivalence relation.

Problem 5 Let $\gamma : [a,b] \to \mathbf{R}^2 \setminus \{P\}$ be a continuous path, and let ν be a vector in the plane. Let $\gamma + \nu$ denote the path in $\mathbf{R}^2 \setminus \{P + \nu\}$ defined by $(\gamma + \nu)(t) = \gamma(t) + \nu$. Prove that

$$W(\gamma + \nu, P + \nu) = W(\gamma, P).$$