Problem 1. On $\mathbf{R}$ define the equivalence relation $x \sim y$ if and only if $x - y$ is an integer. Let $\mathbf{T}$ denote the quotient space $\mathbf{R}/\sim$, endowed with the quotient topology, and let $p : \mathbf{R} \to \mathbf{T}$ denote the quotient map. Prove that the mapping $f : \mathbf{R} \to \mathbf{T} \times \mathbf{T}$ given by $f(x) = (p(x), p(x\sqrt{2}))$ is injective, is continuous, but it is not a homeomorphism of $\mathbf{R}$ onto $f(\mathbf{R})$.