¶ **1.** If U is an open set and F is closed set in a topological space $(X, \mathcal{T})$, then $U \setminus F$ is open and $F \setminus U$ is closed. (Recall that if $A, B \subset Y$, then $A \setminus B = \{y \in Y \mid (y \in A) \,\&\, (y \notin B)\}$.)

¶ **2.** Let X be the real numbers with the topology which has the sets (a, ∞), $a \in \mathbf{R}$, as a base. Which sequences converge to which points? What is the closure of $(-\infty, 0)$?

¶ **3.** (Sorgenfrey line) Show that the sets $[a, b)$, a, b real numbers, form a base for a topology on the real line. Determine which of the following subsets of X are open and which ones are closed: $(-\infty, a)$, $[a, b)$, $[a, \infty)$, (a, b), (a, ∞), $(-\infty, a]$, $[a, b]$, $\{a\}$.

¶ **4.** Let X be the set of integers with the cofinite topology: $U \subset X$ is open if $X \setminus U$ is finite or equal to X. Prove that the sequence $\{1, 2, 3, \cdots\}$ converges to all points of X. What are the convergent sequences in X.

¶ **5.** Let X be a set and let $\mathcal{T}$ be the collection of all subsets U in X such that $X \setminus U$ is at most countable or equal to X.

(a) Prove that $\mathcal{T}$ is a topology on X.

(b) Describe all convergent sequences in X.

(c) Let X be the set or real numbers. Find a subset S of X an a point in the closure of S that is not the limit of any sequence of points in S.

¶ **6.** Let X be the set of positive integers $n \geq 2$. Show that the sets $U_n = \{x \in X; x \text{ divides } n\}$, $n \geq 2$, form a base for a topology on X. Find the closure of the one-point sets $\{x\}$, $x \in X$, and of the set of prime numbers.

¶ **7.** For each positive integer n, let $S_n = \{n, n+1, \ldots\}$. The collection of all subsets of $\mathbf{N}$ which contain some S_n is a base for a topology on $\mathbf{N}$.

¶ **8.** Prove or disprove:

(a) The intersection of an arbitrary family of topologies on X is a topology on X.

(b) The union of two topologies on X is a topology on X.

¶ **9.** If $\mathfrak{B}$ is a base for a topology on X, then the topology generated by $\mathfrak{B}$ equals the intersection of all topologies on X that contain $\mathfrak{B}$.

¶ **10.** Let X be a set. A closure operator κ on X is a mapping that to each subset A of X assigns another subset A^κ of X and such that it satisfies the following properties:

(a) $\emptyset^\kappa = \emptyset$.

(b) $A \subset A^\kappa$ for all $A \subset X$.

(c) $(A^\kappa)^\kappa = A^\kappa$, for all $A \subset X$.

(d) $(A \cup B)^\kappa = A^\kappa \cup B^\kappa$, for all $A, B \subset X$.

Prove that if κ is a closure operator on X, then there is a unique topology $\mathcal{T}$ on X so that the closure of A in $(X, \mathcal{T})$ is A^κ, for all $A \subset X$.