¶ **1.** Prove that if $H, K \subset X$ are both compact, then $H \cup K$ and $H \cap K$ are also compact.

¶ **2.** Let X be a (non-empty) compact metric space and $f : X \to X$ be continuous. Prove that there is a non-empty subset $A \subset X$ such that $f(A) = A$.

¶ **3.** Let X be a metric space. Prove that a subset S of X is compact (as a metric space with the induced metric) if and only if the following is true: every collection of open subsets of X whose union contains S admits a finite subcollection whose union still contains S.

¶ **4.** (a) Prove that if $A \subset X$ is compact, then A is closed and bounded.

(b) Prove that if $F \subset X$ is closed and X is compact, then F is also compact.

(c) True or false: if $A \subset X$ is closed and bounded, then A is compact.

(d) True or false: if X is complete and if $A \subset X$ is closed and bounded, then A is compact.

¶ **5.** Prove that a metric space in which every closed ball is compact is complete.

¶ **6.** Prove that X and Y are both compact if and only if the product $X \times Y$ is compact.

¶ **7.** Let $f : X \to Y$ be continuous.

(a) Prove that if $K \subset X$ is compact, then $f(K)$ is compact.

(b) Prove that if f is onto and and one-to-one, then $f^{-1} : Y \to X$ is also continuous.

¶ **8.** Prove that a continuous mapping from a compact metric space to a second metric space is uniformly continuous.

¶ **9.** Prove that if X is compact and $\mathcal{U}$ is an open covering of X, then there is $\epsilon > 0$ such that every subset of X of diameter $< \epsilon$ is contained in at least one element of $\mathcal{U}$.

¶ **10.** A family of subsets of a set if said to have the finite intersection property if every finite subfamily has non-empty intersection. Prove that a metric space is compact if and only if every family of closed non-empty subsets that has the finite intersection property has non-empty intersection.