¶ **1.** Prove that for any three points x, z, w in a metric space we have

$$|d(x,w) - d(z,w)| \leq d(x,z).$$

¶ **2.** Prove that in any metric space $d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + \cdots + d(x_{n-1}, x_n)$.

¶ **3.** Let X be a set and $d : X \times X \to \mathbf{R}$ be a function such that $d(x,x) = 0$, and for $x \neq z$, $d(x,z) = d(z,x)$ is a number between 1 and 2 (which may depend on x and z). Prove that d is a metric on X.

¶ **4.** Let X be a set and $d : X \times X \to \mathbf{R}$ be a function that satisfies $d(x,x) = 0$, $d(x,z) \neq 0$ for $x \neq z$, and $d(x,z) \leq d(z,w) + d(w,x)$. Prove that d is a metric on X.

¶ **5.** Let d be a metric on X. Prove that $\dfrac{d}{1+d}$ and $\min\{1, d\}$ are also metrics on X.

¶ **6.** Let X be the set of all sequences $x = \{x_n\}_{n=0}^{\infty}$ of real numbers, and let $d : X \times X \to \mathbf{R}$ be the function

$$d(x,z) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{|x_n - z_n|}{1 + |x_n - z_n|}.$$

Prove that d is a metric on X.

¶ **7.** Let d_1 and d_2 be metrics on X. Which of the following are metrics on X: (a) $d_1 + d_2$; (b) $\max\{d_1, d_2\}$; (c) $\min\{d_1, d_2\}$?

¶ **8.** Let (X, d_Y) and (Z, d_Z) be metric spaces. Prove that the Cartesian product $X \times Z$ is a metric space with

$$d((x_1, z_1), (x_2, z_2)) = \sqrt{d_X(x_1, x_2)^2 + d_Z(z_1, z_2)^2}.$$

¶ **9.** Let X be the set of finite words in an alphabet. For two words x, z in X, let $D(x,z)$ be the minimum number of edit operations needed to transform the word x into the word z, where an edit operation is an insertion, deletion, or substitution of a single character of the alphabet. Prove that D is a metric on X.

¶ **10.** (a) Prove that in any metric space the complement of a point is an open set.

(b) Prove that any set in a metric space is an intersection of open sets.