10-4. The Derivative

(a) [Page 186] The average rate of change of a function \( y = f(x) \) is the ratio of the change in \( y \) to the change in \( x \).

(b) [Page 188] The average rate of change of \( y = f(x) \) form \( x = a \) to \( x = b \) is \( \frac{f(b) - f(a)}{b - a} \). Geometrically, this is the slope of the secant line through the points \((a, f(a))\) and \((b, f(b))\).

(c) [Page 190] The instantaneous rate of change is the limits of the average rate of change as the change in \( x \) approaches 0.

(d) [Page 191] The slope of the tangent line to \( y = f(x) \) at the point \((a, f(a))\) is the limit of the slope of the secant line through the points \((a, f(a))\) and \(a + h, f(a + h)\) as \( h \) approaches 0.

(e) The derivative of \( y = f(x) \) at \( x \), denoted \( f'(x) \), is the limit of the difference quotient \( \frac{f(x + h) - f(x)}{h} \) as \( h \to 0 \) (if this limit exists).

(f) [Page 192] Step-by-step method to find derivatives.

(g) An equation for the tangent line to \( y = f(x) \) through the point \((a, f(a))\) is \( y - f(a) = f'(a)(x - a) \) (slope-point form).

10-5. Basic Differentiation Rules

(a) [Page 199] The derivative of a constant function is 0.

(b) [Page 200] Power rule: the derivative of \( f(x) = nx^{n-1} \) is \( f'(x) = nx^{n-1} \).

(c) [Page 204] If \( k \) is a number and \( f(x) \) has derivative \( f'(x) \), then the function \( kf(x) \) has derivative \( kf'(x) \).

(d) [Page 206] The derivative of the sum or the difference of two functions is the sum or the difference of the derivatives of those functions, respectively.

10-7. Marginal Analysis

(a) [Pages 207-215] If \( C(x) \) is the cost of producing \( x \) items, then \( C'(x) \) is the marginal cost, and \( C(x + 1) - C(x) \) is the exact cost of producing the \((x + 1)\)-th item. The marginal cost \( C'(x) \) is approximately the cost of producing the \((x + 1)\)-th item, that is, \( C'(x) \approx C(x + 1) - C(x) \).

(b) [Page 219] Similar statements can be made regarding the total revenue function \( R(x) \) and the total profit function \( P(x) \).

(c) If \( C(x) \) is the cost of producing \( x \) units, then the average cost, or cost per unit, is \( \bar{C} = \frac{C(x)}{x} \). Similar concepts apply to revenue and profit.

11-3. Product and Quotient Rules for Derivatives

(a) [Page 226] Product rule: If \( f(x) = F(x)G(x) \), then \( f'(x) = F'(x)G(x) + F(x)G'(x) \)

(b) [Page 229] Quotient rule: If \( f(x) = \frac{T(x)}{B(x)} \), then \( f'(x) = \frac{T'(x)B(x) - T(x)B'(x)}{B(x)^2} \)

11-4. The Chain rule

(a) [Page 239] If \( g(x) = (f(x))^n \), then \( g'(x) = nf'(x)f(x)^{n-1} \).

11-7. Elasticity of Demand

(a) [Page 248] The relative rate of change of \( f(x) \) is \( \frac{f'(x)}{f(x)} \).

(b) [Page 252] If demand \( x \) and price \( p \) are related by \( x = f(p) \), then the elasticity of demand is given by \( \frac{-pf''(p)}{f'(p)} \).

(c) [Page 255] The demand is inelastic if \( 0 < E(p) < 1 \) (a % increase in price produces a smaller % decrease in demand), demand is unitary if \( E(p) = 1 \) (a % change in price results in the same % change in demand), and demand is elastic if \( E(p) > 1 \) (a % increase in price results in a larger % decrease in demand).

(d) [Page 268] If \( R(p) = pf(p) \) is the revenue function, then \( R'(p) = 0 \) precisely when \( E(p) = 1 \); that is, demand is unitary when the marginal revenue is zero.

12-5. Absolute Maximum and Minimum

(a) [Page 279] If \( f(c) \geq f(x) \) for all \( x \) in domain of \( f \), then \( f(c) \) is the absolute maximum value of \( f \). If \( f(c) \leq f(x) \) for all \( x \) in domain of \( f \), then \( f(c) \) is the absolute minimum value of \( f \).

(b) [Page 281] If \( f(x) \) is continuous on a closed interval \([a, b]\), then \( f \) has both an absolute maximum value and an absolute minimum value on \([a, b]\).

(c) [Page 282] The critical points of a function \( f(x) \) are the points \( x \) in domain \( f \) where the derivative \( f'(x) = 0 \) or where \( f'(x) \) does not exist.

(d) [Page 283] The steps to find the absolute maximum value and the absolute minimum value of a function \( f(x) \) on an interval \([a, b]\) are the following:

1. Make sure that \( f \) is continuous on \([a, b]\).
2. Find the critical values of \( f \) on \([a, b]\).
3. Find the values \( f(x) \) of \( f \) at the critical values and at the endpoints \( a \) and \( b \).
4. The largest value obtained in step (3) is the absolute maximum value of \( f \) on \([a, b]\). The smallest value obtained in step (3) is the absolute minimum value of \( f \) on \([a, b]\).

12-6. Optimization

(a) [Page 298] Application of absolute maximum and absolute minimum to business and economic models (e.g., max profit).