Math 103. Practice 2nd Midterm. Solutions

Problem 1 Suppose that you deposit $13,000 into an investment account.

(a) What amount, $A$, will your account have after 2 years if it is invested at an annual rate of 10% compounded weekly?

$$A = (13000) \left(1 + \frac{0.10}{52}\right)^{2 \cdot 52} \approx 15,253.76$$

(b) What annual rate of interest is needed in order for the investment account to double in 10 years if interest is compounded continuously? $r = (10 \ln 2)\%$

Problem 2 An investment account earns 5% compounded monthly. An initial investment of $8,000 (present value) grows to $20,000 (future value) in $t$ years. Find $t$.

$$t = \frac{\ln(2.5)}{12 \ln(1 + 0.05/12)} = 18.364 \approx 18\text{yr 4 months 11 days}$$

Problem 3 Suppose we invest $400.

(a) What amount, $A$, will our account have after 3 years if it earns an annual rate of 6% compounded quarterly?

$$A = 400(1.015)^{3 \cdot 4} \approx 478.247$$

(b) How long will it take for our account to grow to $1500 if it is invested at an annual rate of 6% compounded weekly?

$$t = \frac{\ln(1500/400)}{52 \ln(1.015)} = 22.042 \text{ years} \approx 22\text{ yr 2 weeks}$$

Problem 4 A long-distance telephone service charges $0.51 a minute for the first 20 minutes or less of a call and $0.07 per minute for each additional minute or fraction thereof.

(a) Complete the piecewise definition of the charge $F(x)$ for a long-distance call lasting $x$ minutes.

$$F(x) = \begin{cases} S(x), & x \leq a, \\ L(x), & x > a. \end{cases}$$

Where

$$S(x) = 0.51x$$

$$L(x) = 10.20 + 0.07(x - 20)$$

and $a = 20$

(b) Find (write DNE if the limit does not exist): 

(i) $\lim_{x \to 20^-} F(x) = 10.20$

(ii) $\lim_{x \to 20^+} F(x) = 10.20$

(iii) $\lim_{x \to 20} F(x) = 10.20$

Problem 5 For the function $f(x) = \frac{4 - 2x}{10 + 7x}$,

(a) Find the limit $\lim_{x \to \infty} f(x) = \frac{-2}{7}$

(b) Find the limit $\lim_{x \to -10/7^+} f(x) = +\infty$

(c) Find the limit $\lim_{x \to -10/7^-} f(x) = -\infty$

(d) Find all the numbers $x$ where $f$ is continuous. (Write your answer using interval notation.)

$$(-\infty, -10/7) \cup (-10/7, \infty)$$

Problem 6 Let $f$ be the following piecewise function:

$$f(x) = \begin{cases} 2x + 4, & x < 0, \\ 4, & 0 \leq x < 2, \\ -x + 9, & 2 \leq x. \end{cases}$$

(a) Find $\lim_{x \to 0^-} f(x) = 4$

(b) Find $\lim_{x \to 2^+} f(x) = 7$

(c) Where is this function continuous? Write your answer using interval notation. $(-\infty, 2) \cup (2, \infty)$

Problem 7 For the function $f(x) = \frac{2x^2 - 7}{x^2 - 9x + 14}$ find the limits (if they exist; if they do not exist, write DNE).

(a) $\lim_{x \to 7} f(x) = \frac{13}{5}$

(b) $\lim_{x \to 2} f(x) = $ DNE

(c) $\lim_{x \to 1} f(x) = -1$

Problem 8 The (heuristic) Rule of 72 states that if an investment earns $r\%$ year, it will take approximately $72/r$ years for your money to double.

You invest $7500 at a 2% interest rate, compounded annually.

(a) According to the rule of 72, what is the doubling time, in years, for this investment?

$$36 \text{ years}$$

(b) According to the Rule of 72, how much money will your investment be worth after 180 years?

$$\$240,000$$

(c) Use the compound interest formula to find how much the investment will be worth after 180 years.

$$A = 7500(1.02)^{180} \approx \$264,906$$