Math 103. Practice 2nd Midterm

Problem 1  Suppose that you deposit $13,000 into an investment account.

(1) What amount, A, will your account have after 2 years if it is invested at an annual rate of 8% compounded weekly?

(2) What annual rate of interest is needed in order for the investment account to double in 10 years if interest is compounded continuously?
Problem 2 An investment account earns 5% compounded monthly. An initial investment of $8,000 (present value) grows to $20,000 (future value) in \( t \) years. Find \( t \).

Problem 3 Suppose we invest $400.

(1) What amount, \( A \), will our account have after 3 years if it earns an annual rate of 6% compounded quarterly?

(2) How long will it take for our account to grow to $1500 if it is invested at an annual rate of 6% compounded weekly?
Problem 4 A long-distance telephone service charges $0.51 a minute for the first 20 minutes or less of a call and $0.07 per minute for each additional minute or fraction thereof.

(1) Complete the piecewise definition of the charge $F(x)$ for a long-distance call lasting $x$ minutes.

$$F(x) = \begin{cases} 
  S(x), & x \leq a, \\
  L(x), & x > a.
\end{cases}$$

Where

$S(x) =$

$L(x) =$

and $a =$

(2) Find (write DNE if the limit does not exist):

(i) $\lim_{x \to 20^-} F(x)$

(ii) $\lim_{x \to 20^+} F(x)$

(iii) $\lim_{x \to 20} F(x)$
Problem 5  For the function \( f(x) = \frac{4 - 2x}{10 + 7x} \),

(1) Find the limit \( \lim_{x \to \infty} f(x) \).

(2) Find the limit \( \lim_{x \to -\frac{10}{7}^+} f(x) \).

(3) Find the limit \( \lim_{x \to -\frac{10}{7}^-} f(x) \).

(4) Find all the numbers \( x \) where \( f \) is continuous. (Write your answer using interval notation.)
Problem 6 Let $f$ be the following piecewise function:

\[
    f(x) = \begin{cases} 
        2x + 4, & x < 0, \\
        4, & 0 \leq x < 2, \\
        -x + 9, & 2 \leq x. 
    \end{cases}
\]

(1) Find $\lim_{x \to 0^+} f(x)$

(2) Find $\lim_{x \to 2^-} f(x)$

(3) Where is this function continuous? Write your answer using interval notation.
Problem 7  For the function \( f(x) = \frac{2x^2 - 15x + 7}{x^2 - 9x + 14} \) find the limits (if they exist; if they do not exist, write DNE).

1. \( \lim_{x \to 7} f(x) \)

2. \( \lim_{x \to 2} f(x) \)

3. \( \lim_{x \to 1} f(x) \)
Problem 8 The (heuristic) Rule of 72 states that if an investment earns \( r \% \) year, it will take approximately \( \frac{72}{r} \) years for your money to double.

You invest 7500 at a 2\% interest rate, compounded annually.

1) According to the rule of 72, what is the doubling time, in years, for this investment?

2) According to the Rule of 72, how much money will your investment be worth after 180 years?

3) Use the compound interest formula to find how much the investment will be worth after 180 years.