Math 102. Key Concepts for Chapter 3 Quiz

A. Basic Simplification

Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for a polynomial function.

Problem 1 For $H(x) = 3 - 4x - 4x^2$, find $\dfrac{H(x+h)-H(x)}{h}$ and simplify completely.

B. Find the domain of a function

Problem 2 Find the domain of $f(x) = \dfrac{4}{\sqrt{x-3}}$

C. Graphs of basic shapes and piecewise-defined functions

Problem 3 (a) Sketch the graph of $f(x) = \sqrt[3]{x}$

(b) Sketch the graph of

$$f(x) = \begin{cases} x+2, & x \le -1 \\ 1-x^2, & x > -1 \end{cases}$$

D. Graphs of Functions

Sketch the graph of a function using basic shapes combined with transformations.

Problem 4 Sketch the graph of each function, not by plotting points, but by applying transformations to basic shapes:

(a) $f(x) = 2 - \sqrt{3-x}$

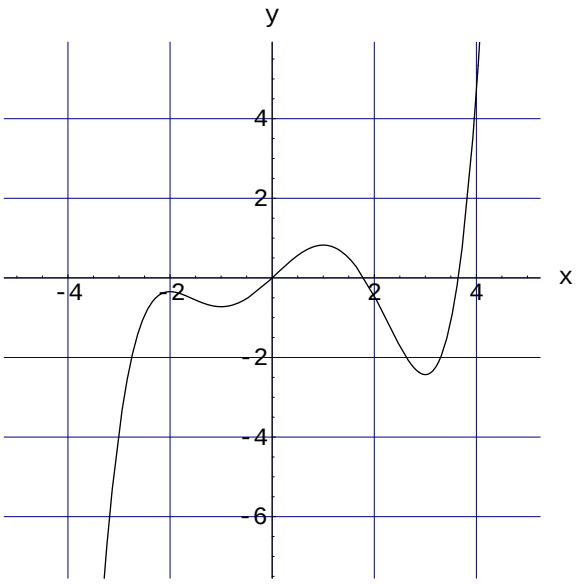
(b) $g(x) = 3 - \left|\dfrac{-x}{2}\right|$

(c) $h(x) = \dfrac{2}{x-1} + 3$

E. Increasing and decreasing. Local maxima and minima. Average rate of Change

Find the average rate of change of a function on a specified interval. Identify the intervals where a function is increasing and the intervals where a function is decreasing. Determine the local maxima and local minima.

Problem 5 The graph of f is given in the figure below.



(a) Find the average rate of change of f between the points $x = -3$ and $x = 1$.

(b) Find the x and y intercepts.

(c) Determine the intervals on which f is increasing and the intervals on which f is decreasing.

(d) Find the local maximum values and the local minimum values of f.

F. Combining functions

Find the sum, difference, product and quotient of two given functions and their domains.

Problem 6 For the functions

$$f(x) = \frac{1}{x+3} \quad \text{and} \quad g(x) = \frac{x}{x-2}$$

(a) Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$ and $(f/g)(x)$.

(b) Find the domains of $f+g$, $f-g$, $f \cdot g$ and f/g.

G. Modeling with functions

Construct and analyze functions from real-life data.

Problem 7 An open box is to be made from a flat square piece of cardboard of 12 inches in length and width by cutting equal squares of length x from each corner and folding up the sides.

(a) Express the volume, V, of the box as a function of x.

(b) What is the domain of V?

Problem 8 A piece of wire 10 feet long is cunt in two pieces. One piece will be shaped as a square, and the other piece will be shaped as a circle. Express the total area, A, enclosed by the two pieces of wire as a function of the length, x, of the side of the square.