

Math 102. Fall 2006. Practice 2nd Midterm

1 Solve $\frac{x}{x-1} \leq \frac{1}{x}$. Write your answer using interval notation.

Solution. $(0, 1)$

2 Let $P(x) = 2x^3 - 5x^2 - 4x + 3$.

- (i) List all the possible rational zeros of P .
- (ii) Verify that 3 is a zero of P .
- (iii) Find all other zeros of P .
- (iv) Find the complete factorization of P .

Solution. (i) $\pm 1, \pm 1/2, \pm 3, \pm 3/2$
 (iv) $2(x-3)(x+1)(x-1/2)$.

3 Find a fourth degree polynomial, $P(x)$, with real coefficients which has zeros $1 + 3i$ and -1 , with -1 a zero of multiplicity 2, and such that its constant coefficient is 20.

Solution. $2x^4 + 14x^2 + 36x + 20$

4 Find a polynomial P with real coefficients that has degree 4, zeros i and $2i$, and whose graph passes through the point $(1, 5)$.

Solution. $\frac{1}{2}x^4 + \frac{5}{2}x^2 + 2$

5 Given that the complex number $2 + i$ is a zero of the polynomial $P(x) = 2x^4 - 8x^3 + 28x^2 - 72x + 90$, find

- (i) all the zeros of P ;
- (ii) the complete factorization of P .

Solution. (ii) $2(x-3i)(x+3i)(x-2-i)(x-2+i)$

6 For the functions

$$f(x) = \frac{1}{x+3} \quad \text{and} \quad g(x) = \frac{1}{x-2}$$

- (i) Find $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$ and $(g \circ g)(x)$
- (ii) Find the domains of $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$.

Solution.

$$\begin{aligned} f \circ g(x) &= \frac{x-2}{3x-5}, & \text{Domain: } x \neq 2, \frac{5}{3} \\ g \circ f(x) &= \frac{x+3}{-2x-5}, & \text{Domain: } x \neq -3, -\frac{5}{2} \\ f \circ f(x) &= \frac{x+3}{3x+10}, & \text{Domain: } x \neq -3, -\frac{10}{3} \\ g \circ g(x) &= \frac{x-2}{5-2x}, & \text{Domain: } x \neq 2, \frac{5}{2} \end{aligned}$$

7 Let $f(x) = 1 - (x+2)^3$.

- (i) Use transformations and basic shapes to sketch the graph of f .
- (ii) Prove that $f(x)$ is one-to-one.
- (iii) Find an equation for the inverse function f^{-1} of f .
- (iv) Sketch the graph of f^{-1} and the graph of f (you did this in part (i)) in the same coordinate axes.

Solution. $f^{-1}(x) = -2 + \sqrt[3]{1-x}$

- 8 (i) If $f(x) = 2 - \sqrt{3-x}$, find the inverse function f^{-1} .
 (ii) Sketch the graphs of f and f^{-1} on the same coordinate axes.

9 For the function $f(x) = 4 - e^{-x}$.

- (i) Graph f using transformations. State domain, range, intercepts, and horizontal asymptote.
- (ii) Determine the inverse f^{-1} . State domain, range, intercepts, and vertical asymptote.
- (iii) Graph g and g^{-1} on the same pair of coordinate axes.

10 Sketch the graph of $f(x) = 3 - \log(1-x)$. Label intercepts and asymptotes.

11 Simplify the following expression (write as a single logarithm): $20 \log_2 \sqrt[4]{x} + \log_2(4x^3) - \log_2 4$.

12 Solve for x : $\log_3 x + \log_3(x+3) = 4$.

Solution. $-2 + \sqrt{85}$

13 Solve for x : $e^{2x} + 3e^x - 4 = 0$.

Solution. 0

14 It has been estimated that the rat population of a major city follows an exponential growth. The population triplicated in size during an 18 month period. If the rat population is currently 10 million, what will it be 2 years from now?

Solution. $80/3$

15 At the beginning of an experiment, a scientist has 364 grams of radioactive goo. After two hours, her sample has decayed to 24 grams.

- (i) What is the half life (in minutes) of the goo?
- (ii) Find a formula for $G(t)$, the amount of goo remaining t minutes after the experiment started.
- (iii) How many grams of goo will remain after 14 minutes?