Math 102. Fall 2006. Practice Final Exam

1 For $f(x) = 1 - 7x - 3x^2$, find

(a) $f(a)$;

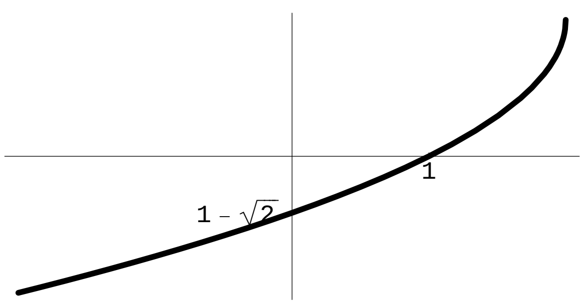
(b) $f(a+h)$;

(c) $\dfrac{f(a+h) - f(a)}{h}$, and simplify completely.

Solution. (a) $\boxed{1 - 7a - 3a^2}$; (b) $\boxed{1 - 7(a+h) - 3(a+h)^2}$; (c) $\boxed{-7 - 6a - 3h}$

2 Use transformations to sketch the graph of $f(x) = 1 - \sqrt{2 - x}$.

Solution.



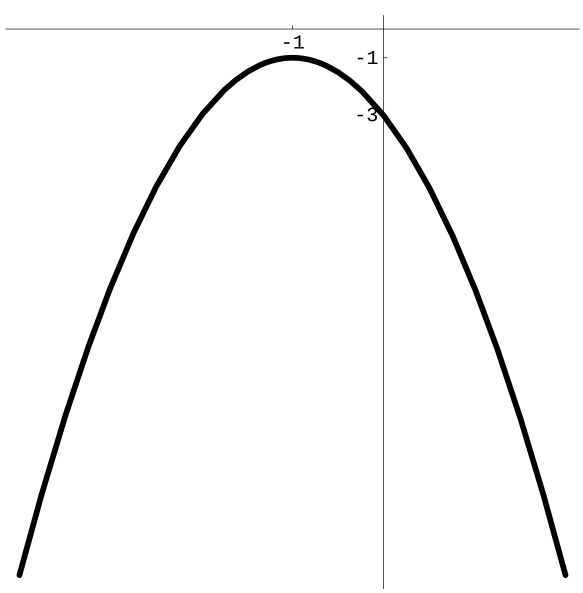
3 For the quadratic function $f(x) = -2x^2 - 4x - 3$:

(a) find the vertex and intercepts;

(b) sketch its graph;

(c) determine the intervals where it is increasing and the intervals where it is decreasing.

Solution.



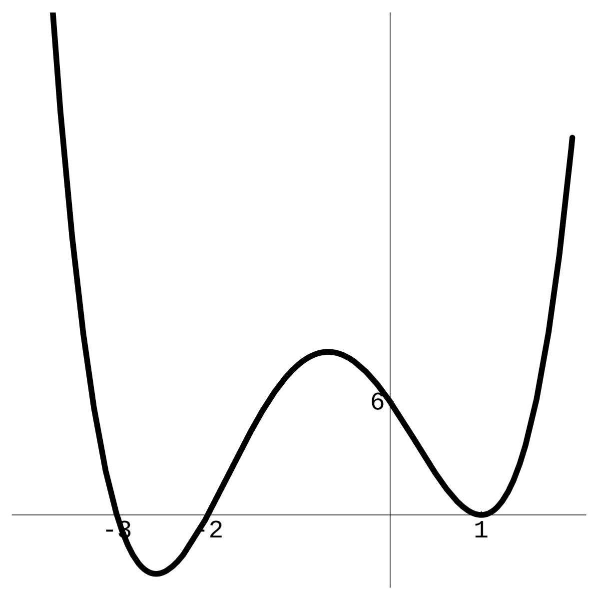
Increasing on $(-\infty, -1]$, Decrasing on $[-1, \infty)$.

4 Paradise Travel Agency's monthly profit P (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent in advertising per month according to the rule $P(x) = 7 - 2x(x - 4)$. What is Paradise's maximum monthly profit?

Solution. $\boxed{\$\ 15{,}000}$

5 Sketch the graph of $f(x) = (x-1)^2(x+3)(x+2)$.

Solution.



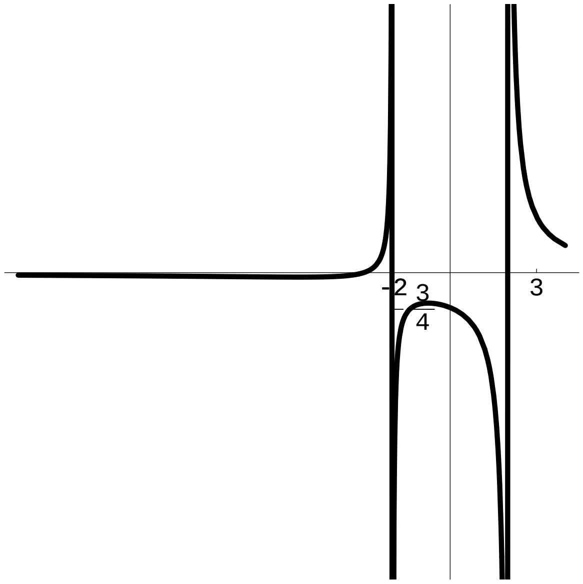
6 For the rational function $R(x) = \dfrac{x+3}{x^2 - 4}$:

(a) Find the vertical asymptotes and the horizontal asymptotes, if any.

(b) Find its intercepts.

(c) Sketch its graph.

Solution. (a) Vertical: $\boxed{x = 2 \text{ and } x = -2}$; Horizontal: $\boxed{y = 0}$
(b) x-int: -3, y-int: $-3/4$; (c)



7 Solve the inequality $\frac{x}{x+2} \le \frac{1}{x}$. Write your answer using interval notation.

Solution. $(-2, -1] \cup (0, 2]$

8 Find all the zeros of the polynomial $P(x) = 2x^3 - 5x^2 + 6x - 2$.

Solution. $\boxed{1/2, 1-i, 1+i}$

9 For $f(x) = \frac{1}{2-x}$ and $g(x) = \frac{3}{x+1}$, find the composite function $f \circ g$ and its domain.

Solution. $\boxed{f \circ g(x) = \frac{x+1}{2x-1}}$, Domain $\boxed{x \ne -1, 1/2}$

10 For $f(x) = \frac{x}{2x-1}$, find

(a) the inverse function $f^{-1}(x)$;

(b) the domain and the range of f.

11 For $f(x) = 1 + e^{-x}$:

(a) find its intercepts and asymptotes;

(b) sketch its graph.

12 For $f(x) = 3 - \ln(x-1)$:

(a) find its intercepts and asymptotes;

(b) sketch its graph.

13 Solve for x: $\log_3 x + \log_3(x-2) = 1$.

14 The population of a colony of mosquitoes follows the law of uninhibited growth. If there are 1000 mosquitoes initially and 1800 after 1 day, what is the size of the colony after 3 days? How long is it until there are 10,000 mosquitoes?

15 Write an equation for the parabola whose focus is $(3, -1)$ and whose directrix is the line $x = 1$.

Solution. $\boxed{4(x-3) = (y+1)^2}$

16 For the ellipse defined by the following equations, determine their centers, major axes, vertices, and foci and sketch their graphs.

(a) $x^2 + 9y^2 - 3 = 0$

(b) $x^2 + 3y^2 - 12y + 9 = 0$

Solution. (a) Center: $(0, 0)$, Vertices: $(\pm sqrt3, 0)$, Foci: $(\pm\sqrt{8/3}, 0)$

17 Find an equation for the ellipse with foci $(0, \pm 4)$, center at the origin, and minor axis with length of 6.

Solution. $\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$

18 For the hyperbolas defined by the following equations determine their centers, vertices, foci, transverse axis, asymptotes and sketch their graphs.

(a) $5y^2 - 7x^2 = 70$,

(b) $y^2 - x^2 + 4x - 4y - 1 = 0$

Solution. (a) Center: $(0, 0)$; Vertices: $(0, \pm\sqrt{14})$; Transverse Axis: y-axis; Foci: $(0, \pm\sqrt{24})$; Asympt. $y = \pm(14/10)x$

19 Find an equation for the hyperbola with foci $(0, \pm 7)$ and vertices $(0, \pm 5)$

Solution. $\boxed{\frac{y^2}{25} - \frac{x^2}{24} = 1}$

20 Find an equation for the hyperbola with foci $(0, \pm 7)$ and asymptotes $y = \pm 2\sqrt{2}x$.

21 Solve the following system of equations by transforming its augmented matrix to reduced row echelon form.

$$\begin{cases} x + y - z = 6 \\ 3x - 2y + z = -5 \\ x + 3y - 2z = 14 \end{cases}$$

Solution. $\boxed{x = 1, y = 3, z = -2}$

22 Solve the system

$$\begin{cases} 3x - y - 10 = 0 \\ x^2 + y^2 - 100 = 0 \end{cases}$$

Solution. $\boxed{(0, -10), (6, 8)}$

23 (a) Graph the system of inequalities

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \ge 2 \\ x + y \le 8 \\ 2x + y \le 10 \end{cases}$$

(b) The inequalities in (a) are the constraints of a linear programming problem. Maximize the objective function $z = 2x + 3y$ subject to those constrains.